

Remembering the Mathematics of the Ideal Villa

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It is now fifty years since Colin Rowe's celebrated essay first revealed what Le Corbusier had concealed about the mathematics of the neo-Palladian structural grid of Villa de Monzie/Stein at Garches (1927)—namely, the ratios of the structural intervals that define the organization of the villa from front to back. The importance of this discovery, reported in Rowe's "The Mathematics of the Ideal Villa: Palladio and Le Corbusier compared" (1947), has been underappreciated in subsequent scholarship, which has focused instead on Rowe's discovery of the accord between Garches and Palladio's Villa Malcontenta (c. 1550–1560) with respect to the so-called ABABA rhythm of the intervals from side to side. However, the north-south intervals are no less essential to a proper apprehension of Le Corbusier's structural/spatial grid as a complete idea. This article reexamines the mathematics of the grid employed at Villa de Monzie/Stein and proposes an alternative to the Le Corbusier-Rowe numbering system. The alternative system heightens perception of the grid's fundamental mathematical elegance and ideality as well as its comprehensive control of the "extended field" of the site and elevations/facades. This study seeks to cast new light on the significance of Le Corbusier's assertion that, at Garches, more than at any of his other projects, "proportion ruled absolutely there, as absolute mistress."

One always has to say what one sees, and especially, which is more difficult, one always has to see what one sees.

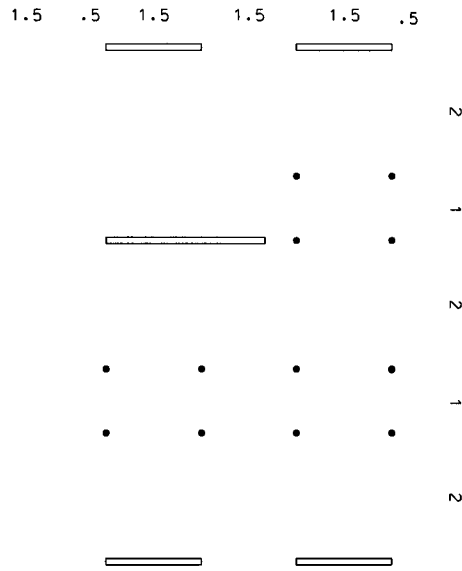
—Le Corbusier¹

THE SO-CALLED ABABA RHYTHM OF THE STRUCTURAL GRID OF LE Corbusier's remarkable Villa de Monzie/Stein of 1927 at Garches enjoys legendary status in the history of modern architecture. However, the celebrated essay considered to be responsible for explicating the villa's neo-Palladian formula, Colin Rowe's "The Mathematics of the Ideal Villa: Palladio and Le Corbusier compared,"² first published fifty years ago, actually made no reference to the structural intervals using the designations *A* and *B*. The popularization of the formulation ABABA is attributable not to Rowe himself, but rather to commentators on his essay.³ In point of fact, following the iconic elevation diagrams published by Le Corbusier in his *Oeuvre complète*,⁴ Rowe only applied the terms *A* and *B* in connection with a related but different mathematical property of the villa—namely, the use of the Greek golden section as a proportional device.⁵ With respect to the rhythm of the structural intervals between the villa's end walls, Rowe was equally strict in employing Le Corbusier's own designations, which are indicated on the same elevation diagrams as "2:1:2:1:2." The numerical sequence, in contrast to the alphabetical one, has the obvious benefit that it not only indicates the alternation of bays but also relates information as to their ratios.

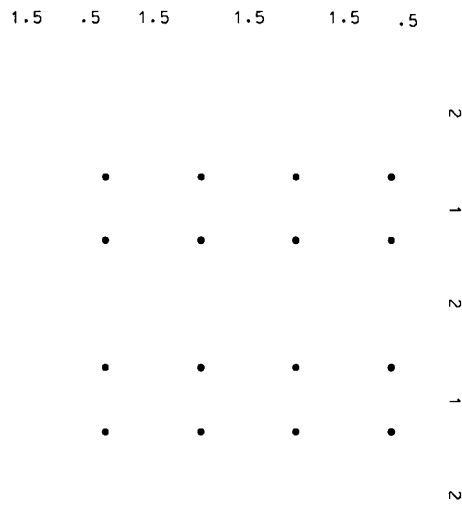
This issue of nomenclature may seem trivial, yet it functions, I believe, as the threshold to a larger debate aimed at heightening perception of the inherent paradigmatic mathematical structure of the Villa de Monzie/Stein, a debate that Rowe's brilliant essay initiated. That is to say, with respect to the central issue of the villa's *grid*, inasmuch as the question of ABABA versus 2:1:2:1:2 is limited to the problem of describing the *east-west* structural intervals, it may also serve to focus attention on a more significant problem: the degree to which the equally important *north-south* structural intervals—and thus the essential conditions of the grid as a whole—have been underappreciated. The association of the villa at Garches with the ABABA or 2:1:2:1:2 rhythm may be firmly established in the educated architectural mind, but how quickly and surely does one recall the numerical—or alphabetical—sequence of the structural intervals that run parallel to the principal axis of the site? In other words, how well does one remember the grid?⁶

The idea of an architecture that is "totally memorable" is characterized by Rowe, quite rightly, as an abstract attribute of no small significance. He introduces the idea in the first sentence of his essay: "As the ideal type of centralized building Palladio's Villa Capra-Rotunda has, perhaps more than any other house, imposed itself upon the imagination. Mathematical, abstract, four square, without apparent function and totally memorable."⁷ On many levels, Le Corbusier's villa at Garches, an iconic example of modern architecture, is also totally memorable. However, the same cannot be said of its celebrated "grid," at least not the form in which it has been mathematically represented to date. I maintain, however, that if it is considered through a new optic, if the mathematical expression is *transformed* or *defamiliarized*, then the grid, rather ironically, is found to be possessed of the essential quality that makes it totally memorable: the quality of the *ideal*.⁸ According to this hypothesis, the intervals that define the "abscissas" (north-south coordinates) of the grid cannot fail to present themselves as forcibly and enduringly to the mind as the intervals that define the "ordinates" (east-west coordinates), for their mathematical interdependence is shown to be undeniably lucid.⁹

In this article, I propose a simple alternative numbering system for the grid of the Villa de Monzie/Stein at Garches. Ultimately, the proposal deconstructs the authority of the association of the grid with the simplistic and reductive ABABA/2:1:2:1:2 expressions. In addition to revealing the grid's intrinsic ideality—its nature as an elegant mathematical paradigm and, accordingly, its mnemonic simplicity—the alternative numbering system also illuminates other fundamental properties of the villa's proportional substructure, including the composition of the underappreciated end elevations, the site plan, and the main facades.¹⁰



1976



1947

1. Rowe's diagrams, redrawn by the author, of the schematic interval structure of the Villa de Monzie/Stein's plan, based on Rowe's original illustration for the 1947 article (ground floor) and on the revised illustration published in the 1976 edition (*piano nobile*). In the Le Corbusier-Rowe numbering system, the ratios that describe the primary rectangular field of the villa (east-west/north-south) are 2:1:2:1:2:1½:1½:1½:½. The "summary sequence" is ½:1:1½:2.

1: Plan

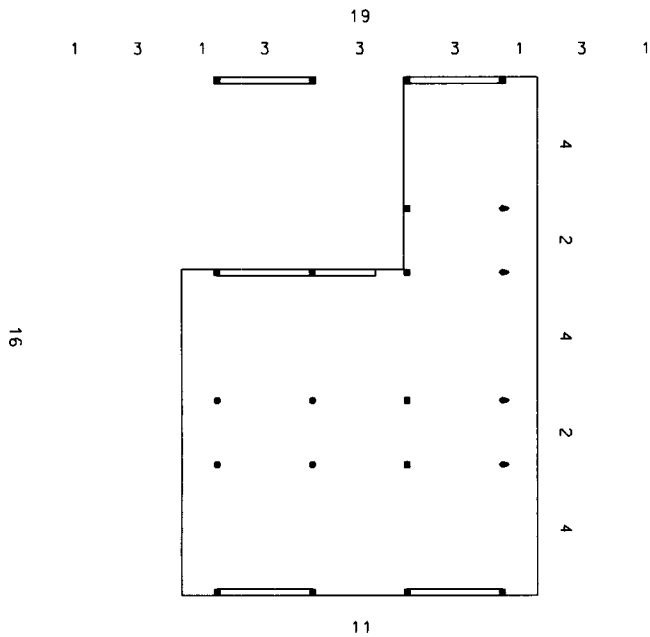
The universe of Platonic and Pythagorean speculation was compounded of the simpler relationships of numbers, and such a cosmos was formed within the triangle made by the square and the cube of the numbers 1, 2, 3. . . . And if such numbers governed the works of God, it was considered fitting that the works of man should be similarly constructed, that a building should be a representative, in microcosm, of the process exhibited at a larger scale in the workings of the world.

—Colin Rowe¹¹

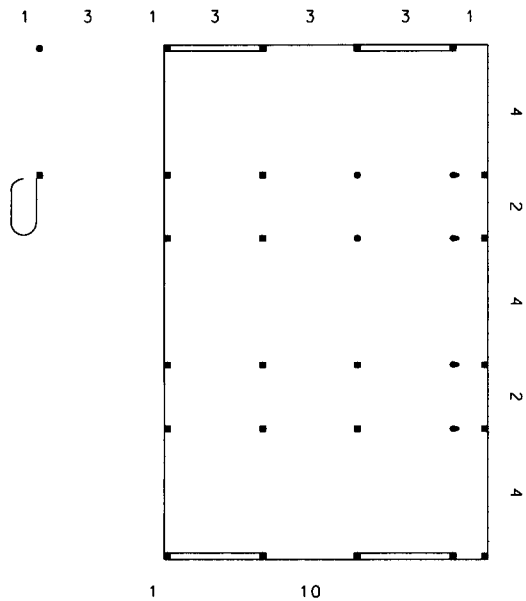
Conceal\Reveal

Rowe's famous analytic diagrams of the Villa de Monzie/Stein and the Villa Malcontenta, which first appeared in print in 1947, illustrated the schematic interval-structure of Le Corbusier's villa by employing what is ostensibly its ground-floor plan. In the 1976 edition of the article, Rowe illustrated the same schematic interval-structure by employing the villa's *piano nobile* plan. I have re-created both diagrams (Figure 1), which may be compared with Le Corbusier's corresponding plans.¹² One of the significant differences between the east-west intervals and the north-south intervals is that the east-west intervals are contained by the building's primary rectangular field of enclosure, whereas the north-south intervals extend into the site and function to organize the spatial relationships of various secondary and tertiary phenomena (for example, the projected south terrace that Rowe includes in his diagram, to which he assigns the interval designation 1½).¹³ This underscores the all-important organizing function of the north-south axis of the site. With this difference in mind, it is still important to consider the grid in the simplest terms as the relationship between the five east-west intervals and the five *major* north-south intervals that together describe the building's primary rectangular field of enclosure. Thus, in the Le Corbusier-Rowe numbering system, the east-west sequence is 2:1:2:1:2, and the major north-south sequence is ½:1½:1½:1½:½. What I call the "summary sequence"—the numbers in ascending order that represent the four different bay sizes or regulating intervals that Le Corbusier used in the villa—is ½:1:1½:2. These are the four numbers that are now associated in the literature of architecture with the villa's fundamental numerical structure.

Le Corbusier was clearly sensitive to what mathematicians call "elegance" with reference to an aesthetic property of mathematical assertions. Thus, it is not surprising that he celebrated the 2:1:2:1:2 proportional sequence of the east-west intervals in the *Oeuvre complète*.¹⁴ According to Rudolf Wittkower, the Pythagorean-Platonic tradition regards the 1:2 ratio, which is the



PIANO NOBILE



GROUND FLOOR

2. Diagrams of the schematic interval-structure of the Villa de Monzie/Stein's ground-floor and *piano nobile* plans, showing the alternative numbering system based on doubling. Ratios that describe the primary rectangular field of the villa (north-south/east-west) are 1:3:3:3:114:2:4:2:4. The "summary sequence" is 1:2:3:4. Column locations and shapes are specific, and additional north-south intervals that extend into the site are also shown.

ratio of the square to the double square (the point of departure for Le Corbusier's later work on the Modulor), as the basis for all musical consonance: "Perfection and beauty were there ascribed to the ratio itself."¹⁵ Ultimately, Greek ideals of mathematical perfection and beauty value whole-number relationships. It is not surprising, therefore, that Le Corbusier chose to suppress the proportional sequence of the fractional north-south intervals, which are not identified in the *Oeuvre complète*. Nor were the end elevations to which they pertain published (though presumably for reasons that have to do with promoting the north and south facades as the primary architectural events of the vertical field).¹⁶ In point of fact, one of the most original aspects of Rowe's essay was that he drew attention to the ratios of the north-south intervals and, in so doing, revealed what Le Corbusier had concealed—namely, the complete mathematical structure of the grid. However, the inelegance of the north-south sequence that Rowe revealed ($\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:1\frac{1}{2}:\frac{1}{2}$) presents no small challenge to one's ability to remember it, and consequently, to remember the grid as a whole.¹⁷

Mathematical Shift/1:2:3:4

Yet, as Rowe may have intended the reader to infer from his diagram, through the simple mathematical device of doubling the numbers, the inelegant fractions are eliminated (Figure 2). According to this alternative numbering system, with respect to the five major intervals that describe the grid in each dimension, the east-west sequence is 4:2:4:2:4, and the north-south sequence is 1:3:3:3:1. The summary sequence is 1:2:3:4. This alternative numbering system clarifies the intrinsic elegance of the whole-number relationships of Villa de Monzie/Stein's mathematical structure. The grid is now seen to be ordered by four significant numbers: 1, 2, 3, and 4.¹⁸

My alternative diagrams depart from Rowe's in other ways as well. They include the suspended entrance canopy to the north and the extra interval of the terrace to the south, from which the outdoor stairway descends to the garden.¹⁹ These additions heighten awareness of the degree to which Le Corbusier regulated the overall plan through the use of the ratios of the first four integers. According to this numbering system, then, the primary rectangular volume of the villa oscillates between two readings: (1) at the ground floor, the ratio of the rectangular field is 10:16; (2) at the *piano nobile*, the ratio of the rectangular field is 11:16. Thus, Le Corbusier defines the limits of an extended horizontal rectangular field whose ratio is 19:16.

These alternative diagrams also represent a fine-tuning of the villa's structural diagram with respect to the realities of the columnar order.²⁰ They reveal the difference between the circumstantial disposition of structure at the "profane" ground floor (thirty-one

columns) and the more idealized disposition of columns at the “sacred” *piano nobile* (twenty-two columns). Significantly, neither of these floors nor the two upper floors exhibit the ideal condition of twenty-four columns that Le Corbusier’s three-bay-by-five-bay structural matrix would imply—that is, the condition sans cantilevered bays. The five ground-floor columns coplanar with the north facade represent an especially significant example of Le Corbusier’s circumstantial deployment of columns. Their presence seems to suggest that the corresponding area of the *piano nobile* above is actually cantilevered only at the northwest corner.²¹

Pythagorean Purism

Rudolf Wittkower, on whose scholarship Rowe relies, wrote that “all systems of proportion in Western art and architecture . . . are ultimately derived from Greek thought. Pythagoras, living in the sixth century B.C., is credited with the discovery that the Greek musical scale depends on the division of a string of the lyre in the ratios 1:2 (octave), 2:3 (fifth), 3:4 (fourth) and 1:4 (double octave). In other words, *the ratios of the first four integers 1:2:3:4 express all the consonances of the Greek musical scale* [emphasis added].”²² As such, the grid of the villa at Garches, based on the ratios of the first four integers 1:2:3:4, constitutes an architectural expression of the fundamental relationships of western musical harmony. Far from being inelegant, Le Corbusier’s spatial partitioning reveals itself to be truly in accord with Rowe’s description of the Greek ideal: a “universe . . . compounded of the simpler relationships of numbers.”²³

In another text, Wittkower explains that, in addition to the octave, fifth, fourth, and double octave, also inherent in the ratios 1:2:3:4 is a fifth ratio, the octave plus a fifth (1:3).²⁴ Moreover, according to Wittkower, three of the five ratios comprise “simple” consonances and two comprise “compound” consonances. The simple consonances include the ratios of the octave (1:2 or 2:4), the fifth (2:3), and the fourth (3:4). The compound consonances include the double octave (1:4) and the octave plus a fifth (1:3). Thus, at Garches, the fundamental ratio of the east-west dimension of the grid is the “simple” consonance 2:4, and by contrast, the fundamental ratio of the north-south dimension of the grid is the “compound” consonance 1:3. The other three ratios function to mathematically interconnect the two dimensions of the grid—in other words, they mediate the fundamental opposition between the east-west and north-south intervals that is expressed as the “simple/compound” consonance, 2:4|1:3.

The chart that follows shows the five ratios of the Greek musical system (including the two different expressions of the octave, 1:2 and 2:4) and their corresponding relationship to the structural/spa-

tial intervals of Le Corbusier’s grid of the villa at Garches. It suggests that, to appreciate fully the villa’s harmonic schema, the “cross-ratios” that interconnect the east-west and north-south intervals (for example, 2:3, the ratio of the *minor east-west* interval to the *major north-south* interval) are just as significant as the ratios that pertain to one direction of the grid only (that is, 2:4 and 1:3). In other words, the chart reveals other aspects of the east-west/north-south interdependence that are implied by the grid’s ideal 1:2:3:4 ratio system.

Villa de Monzie/Stein: Harmonic Ratios of the 1:2:3:4 Grid

| | | | | | |
|-----|----------------|----------|-----------------------|---------------|-----|
| 1:2 | Octave | Simple | Minor n-s : minor e-w | 1.25m : 2.50m | A:B |
| 1:3 | Octave + fifth | Compound | Minor n-s : major n-s | 1.25m : 3.75m | A:C |
| 1:4 | Double octave | Compound | Minor n-s : major e-w | 1.25m : 5.00m | A:D |
| 2:4 | Octave | Simple | Minor e-w : major e-w | 2.50m : 5.00m | B:D |
| 2:3 | Fifth | Simple | Minor e-w : major n-s | 2.50m : 3.75m | B:C |
| 3:4 | Fourth | Simple | Major n-s : major e-w | 3.75m : 5.00m | C:D |

n-s = north-south; *e-w* = east-west; *m* = meters; *A* = 1

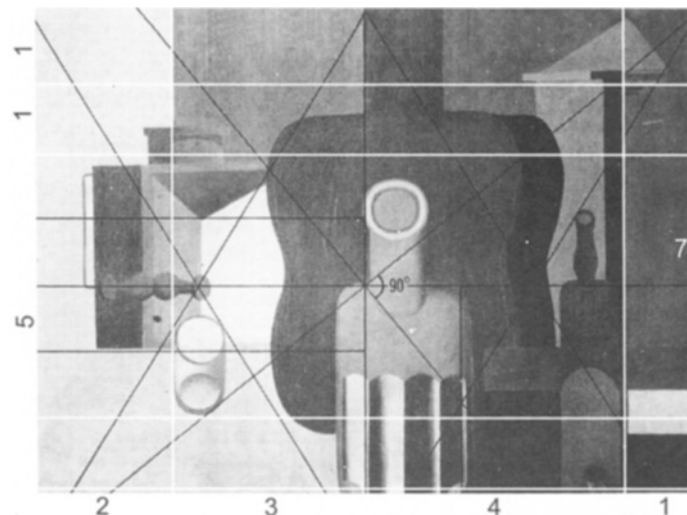
On another level, Wittkower explains the difference between *ratios* (relation of two numbers) and *proportions*, which he describes as “the equality of ratios between two pairs of quantities.”²⁵ Moreover, Wittkower distinguishes between “geometrical proportion,” 1:2:4 (“the first term is to the second as the second is to the third”),²⁶ and “arithmetic proportion,” 2:3:4 (“the second term exceeds the first by the same amount as the third exceeds the second”).²⁷ For example, “If you have two strings under identical conditions, one exactly half the length of the other, and strike them, the pitch of the shorter string is one octave above that of the longer one, i.e., the ratio 1:2 corresponds to the pitch of an octave. By halving the shorter string we get an octave above the first one, and the ratios of the two octaves can be expressed as 1:2:4.”²⁸

Thus, at the Villa de Monzie/Stein, the minor/short north-south interval (the cantilever, 1) and the two east-west intervals (2 and 4) comprise a geometric proportion, 1:2:4, which represents the octave and double octave. That is, in addition to the octave relationship between the minor and major east-west intervals (2:4), the minor north-south interval (the cantilever) is in an octave relationship to the minor east-west interval (1:2) and also in a double-octave relationship to the major east-west interval (1:4). Inherent in the villa’s 1:2:3:4 Pythagorean schema, then, is the double assertion of both ratios and geometric and arithmetic proportions, which combine to signify an ideal system on many levels (including the regulation of the north and south facades, as I discuss below). As Wittkower writes, “The discovery that all musical consonances are arithmetically expressible by the

ratios of the first four integers, the discovery of the close correlation of sound, space (length of the string), and numbers must have left Pythagoras and his associates bewildered and fascinated, for they seemed to hold the key which opened the door to unexplored regions of universal harmony."²⁹ Considered through this optic, then, the structural grid of the villa at Garches is a finely tuned Pythagorean instrument. On one level, it provides an ideal system of order for Le Corbusier's circumstantial game of adding and subtracting columns, and an ideal field for his play of spatial "extension" (expressed through the north-south intervals) and spatial "stasis" (an attribute of the east-west intervals), thus sustaining the dialectic of *fact* versus *implication* with respect to the villa's assertion of the cantilever.³⁰ On another level, the grid functions as the deep conceptual structure underlying Le Corbusier's plastic expression of Significant Form.³¹ Its 1:2:3:4 ratio system provides an idealized structural field in which Le Corbusier interposes, and against which he counterposes, a system of contingent nonstructural figures and subfigures—the "musical" forms—of his Purist architectural fugue. In point of fact, Le Corbusier's purist paintings, such as *Composition with Guitar and Lantern* of 1920 (Figure 3), which functioned as the laboratory for his research into form, manifest the same theme of "consonanceltension" between an underlying 1:2:3:4-based mathematical ordering system and the melodic visual arrangement of figurative forms regulated by his painterly eye. In this light, the presence of musical instruments in Le Corbusier's paintings may be viewed as having deeper significance. The guitar, for example, in addition to various conventional meanings—art, body, solid form, landscape—also signifies the presence of an underlying Pythagorean-Platonic musical/mathematical order. For Le Corbusier, mathematics functioned not only as a civilizing force, as regulator of visual and organizational phenomena, but also as mediator between his architectural mind and his musical, painterly intelligence.³²

Paradigm

Ultimately, the grid of the Villa de Monzie/Stein represents an elegant mathematical paradigm in the classical Greek sense. As such, it is difficult to avoid calling to mind the Parthenon, that "pure creation of the mind,"³³ the symbol of mathematically regulated perfection that Le Corbusier revered so ardently above all other buildings. Thus, if Le Corbusier's employment of the device of the Greek golden section is already central to an understanding of the geometric proportions inherent in the villa at Garches,³⁴ perhaps we may extend this connection in order to associate the ideal—or unforgettable—mathematics of the villa's 1:2:3:4 grid at least as much with the Greek musical/mathematical *taxis*³⁵ as with the ABABA rhythm of Palladio's Italian villa.



3. Le Corbusier, *Composition with Guitar and Lantern*, 1920. (Courtesy of Fondation Le Corbusier.) In addition to geometrical relationships indicated by Le Corbusier's own regulating lines, white lines and numbers that have been added by the author show the 1:2:3:4-based whole number ratio system that underlies the Pythagorean-Purist visual field.

2: End Elevations

Proportion ruled absolutely there, as absolute mistress.

—Le Corbusier³⁶

Reversal \ Displacement—1/A

The significance of this assertion by Le Corbusier about his villa at Garches is underscored by the various implications of the alternative numbering system. In particular, the degree to which the *north-south* intervals regulate important aspects of the composition emerges. This is a point that Le Corbusier's axonometric studies, which permit rare glimpses of the end elevations, illuminate—a point that might well be seen to spring from sustained consideration of one idea: the cantilever. On this iconic technological-formal device of modern architecture (and the role it plays) turns more than mathematical differences between the two alternative numbering systems. In the end, the two numbering systems differ precisely on the point that is arguably of greatest significance in one's attempt to appreciate the elegance of Le Corbusier's construction and that, in the 1:2:3:4 system, is intrinsically associated with the cantilever—that is, the module. With respect to the problem of which structural interval of the grid is primary, or generative—that is, which interval ought to be assigned the significant number 1—the two systems clearly differ on

conceptual and pragmatic levels as much as they do on the level of mathematical expression. In the Le Corbusier-Rowe numbering system, the minor east-west interval, 2.5 meters, is the starting point, 1. This 1, however, does not represent A, as one might logically expect, in the putatively equivalent, popular alphabetical expression ABABA. Nonetheless, the formulations that represent the conventional Le Corbusier-Rowe view of the grid may be expressed as follows, east-west/north-south: 2:1:2:1:2½:1½:1½:1½:½ = ABABAICDDDC (B = 1). In the alternative 1:2:3:4 system, the cantilever interval, 1.25 meters, is the starting point, 1. It is the irreducible basic unit, or module, of which the other intervals are whole multiple moduli. Moreover, 1 = A. Thus, the resulting numerical and alphabetical formulations are as follows, north-southeast-west: 1:3:3:3:1¼:2:4:2:4 = ACCCAIDBDBDB.³⁷

Cantilever/Module

Although in the hierarchy of the Le Corbusier-Rowe numbering system the cantilever is downplayed, Rowe's regard for the unique significance of the cantilever as a determining element of the formal properties of the villa at Garches—and as the principal device that differentiates it conceptually and therefore mathematically from Palladio's villa—is clear.³⁸ Moreover, one is caused to consider that, were it not for Le Corbusier's introduction of this disruptive device, the north-south and east-west intervals of the grid might well have been identical.³⁹ Though Rowe never draws an explicit connection between "cantilever" and "module," he hints at their connection in various ways. For example, he used the term *modular grid* in the original 1947 article,⁴⁰ though, significantly, he did not employ the term in the 1976 edition. He also chose for the lead illustration to the original essay Le Corbusier's iconic perspective of the Dom-ino system. Although this illustration is downplayed in the 1976 edition and, as the caption that accompanied it in the 1947 article indicates,⁴¹ Rowe evoked the Dom-ino primarily to make a different point, clearly it may also be read as an implicit assertion of the cantilever as a primary attribute of concrete-frame technology. Moreover, as Eleanor Gregh has explained, the concept of the module is central to the ideology of the Dom-ino system.⁴² In the end, Le Corbusier's compound Dom-ino system at Garches employs the "cantilever/module" as more than an ideal mathematical regulator; it also functions as a practical constructional regulator. One and one-quarter meters equals approximately four feet (4', 1½", to be precise), which is the standard dimensional module undergirding much American architecture, evident in contemporaneous work of Wright and Schindler as well as in construction practice today. The Villa de

Monzie/Stein can therefore be understood on the simplest level as being based on a 4-foot module. Expressed in feet (approximately), the ratio sequences (north-southeast-west) of the structural intervals are 4:12:12:12:4|16:8:16:8:16, and the summary sequence is 4:8:12:16.

In addition to functioning as the mathematical basis for the "universal harmony" of the ideal 1:2:3:4 grid, this 1.25-meter/4-foot module also functions as a dimensional datum. Its double role provides the basis for the systematic structuring of significant compositional moves and for the standardization of the building's components. In other words, on the one hand, the cantilever/module is the basis for Le Corbusier's rigorous mathematical organization of the horizontal field (i.e., the plan), which is apparent in various aspects, such as (1) the width of the "extra" interval of the cantilevered terrace and stair at the back of the villa and the subdivisions of the entrance canopy at the front, as noted above; (2) the projections of the other principal features of the front facade—namely, the central loggia at the attic floor and the balcony of the *piano nobile* above the secondary entrance, which is discussed below; and (3) the structuring of the "extended field," or larger site, also discussed below. On the other hand, it is the basis for his standardization of important building systems. In addition to the structural system, the glazing system is also clearly regulated by this module, as is seen, for example, in the sixteen windows of the *fenêtres en longueur* of the front facade (sixteen windows, at 1.25 meters each, equals 20 meters) and, correspondingly, in the width of the four-part subdivisions of each of the large windows along the garden facade.⁴³

1:4—Edge/Center

It is important to note the significance of the hierarchical tension that exists between the cantilever/module (1) and the major east-west interval (4) in the alternative numbering system. In terms of their real dimensions, the former (1.25 meters) may be equivalent to the base module, but the latter (5 meters) has no less practical or theoretical significance to Le Corbusier's work. In terms of their interval designations, 1 may be numerically rudimentary, but 4 rivals it symbolically and practically. They are physically juxtaposed at the corners of the facades and also conceptually juxtaposed at many levels, which numerous examples reveal.⁴⁴ If the cantilever interval is numerically fractional (1.25) and resides at the edge, the other is numerically whole (5) and asserts itself principally at the center. Research by others has shown that the middle 5-meter bay of the north facade, in particular, was conceptually dominant for Le Corbusier.⁴⁵ This "edgelcenter" opposition between 1 and 4 may be understood

as a “sign” of the dialectical tension between classicism and modernism, which Rowe identified as one of Garches’s transcendent themes. Consistent with this, a fundamental tension exists between the classical, rectangular figure of the facades that are governed by the east-west intervals (2 and 4, the even numbers) and the modernist, idiosyncratic figure of the elevations that are governed by the north-south intervals (1 and 3, the odd numbers).⁴⁶ The latter, however, are no less rigorously defined mathematically, as is shown in the following analysis.

North-South Readings

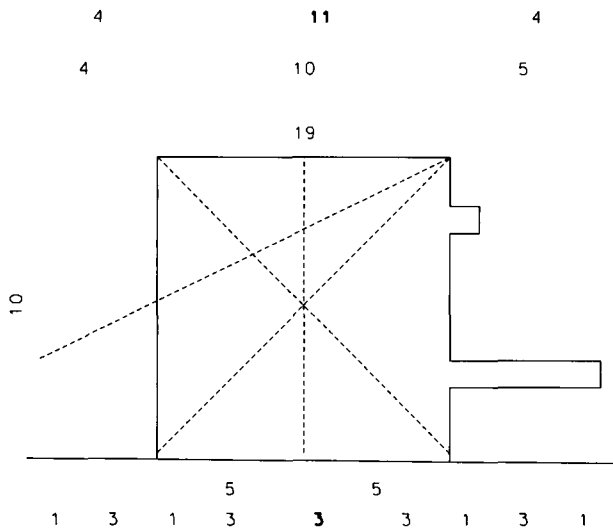
Inasmuch as scholarship (other than Rowe’s) has focused on the east-west intervals of the grid, the end elevations have received little attention. Yet it is clear that the matrix of intervals that regulates their proportional composition (and the corresponding proportional composition of the plan from front to back) is meticulously developed. There is, with respect to the north-south composition, a tension between two overall interval readings, and each reveals a multiplicity of rigorous geometric events.

The first overall reading focuses on the 11-moduli-wide by 10-moduli-high vertical field of the primary volume (Figure 4). The width of this field is defined by the five major structural intervals, 1:3:3:3:1. The windows and various subfigures within the windows, are centered within this field, which implies a centripetal organizational force. The basic symmetry is dimensionally maintained (though a countervailing centrifugal force is introduced) when the extensions to the primary volume—extensions such as the terrace and stair to the south and the suspended entrance canopy to the north—are considered. The extended field is 19 moduli wide by 10 moduli high. The 19 moduli proceed in the sequence 1:3-1:3:3:3:1-3:1, the readings being identical whether read from north to south or from south to north. Within this dimensional symmetry, a subtle, asymmetrical tension is implied by the presence of overlapping 10:10 squares that help to organize architectural events within the 11-moduli-wide by 10-moduli-high vertical field of the main volume. The width of one 10:10 square is defined by the north and south facades of the ground floor. The width of the other 10:10 square is shifted one module to the south to align with the south facade of the *piano nobile* level. These squares set up a tension with the underlying centripetal 4:11:4 emphasis (an emphasis of symmetry) within the overall 19-moduli width of the vertical field. Two centrifugal counterreadings are introduced that establish the presence of tension beneath calm within the plastic structure of the vertical field: 4:10:5 and 5:10:4, reading from north to south along the east or west elevations. Further, the overlapping 10:10 squares help

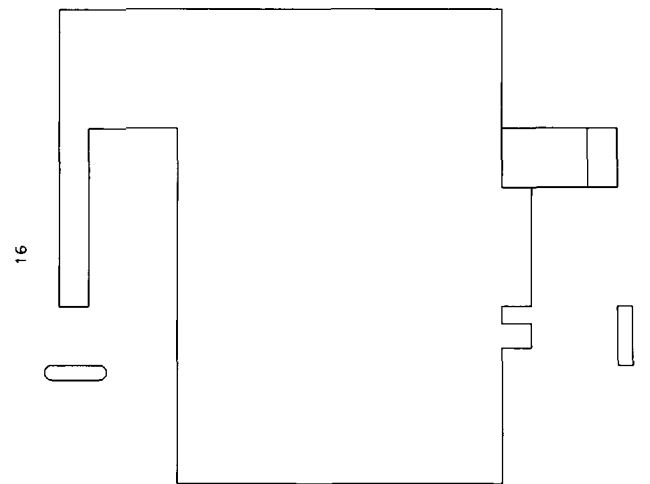
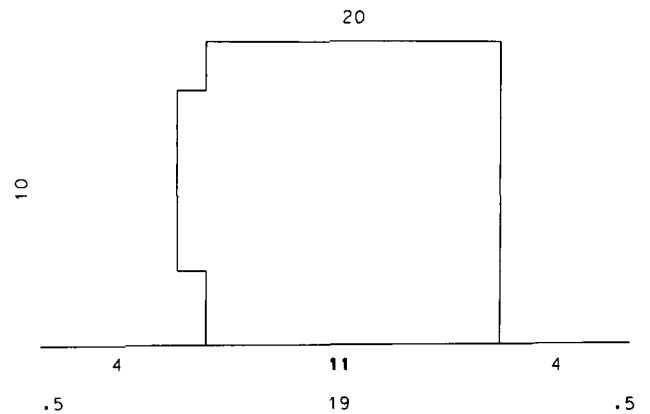
to illuminate the devices of geometric construction that Le Corbusier used to organize the figures and subfigures of the vertical field. Each 10:10 square has a different centerline from the centerline of the 11:10 rectangular field of the principal volume. Le Corbusier has employed all three of these centerlines as interdependent organizing devices. For example, the center of the south-shifted 10:10 square is marked by the north edge of the proposed vent stack on the roof. This is an example of Le Corbusier’s use of visible, physical traces to mark the invisible geometric construct of his fields, a device that he employs in other significant ways at the villa, as is explained later. The vent stack contributes to the system of asymmetrical counterforces that operate within the overall dimensionally symmetrical 19:10 vertical field. The vent stack and terrace pull to the south, and the suspended entrance canopy and balcony projections pull to the north. The diagrams show how the subdiagonals of the overlapping squares govern the integral geometric relationship of the suspended entrance canopy. Within this first overall reading, Le Corbusier provides further evidence of his obsession with proportional rigor in the unexecuted details of his design. One observes his clear intention to extend the 19:16 ratio in plan and the 19:10 ratio in elevation to their more stable, satisfying resolutions (Figure 5). Two minor elements at the northern and southern boundaries that are each equivalent to an additional half module are evident in the axonometric drawings and elevations. Functioning in a way similar to the vent stack on the roof, these two elements mark the geometric limits of a composition that Le Corbusier stretches to 20 moduli from front to back. Thus, significantly, (1) the ratio of depth to height in elevation, 20:10, describes a double square, which is not only important to Le Corbusier as a geometric figure in its own right, but also echoes the 4:2 ratio of the alternating rhythm of the north-south intervals; and (2) the ratio of the depth to width in plan, 20:16, coincides numerically with the metric dimension of the cantilever, the module, 1.25.

Double Cantilever Zone

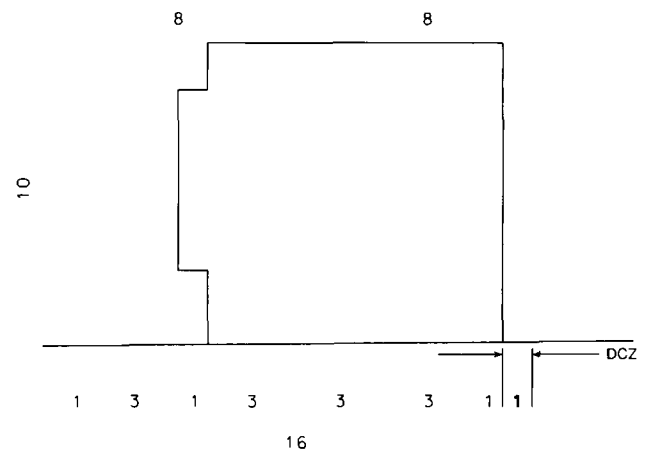
If the first overall interval reading of the east-west composition is defined fundamentally by the boundary of symmetry around the main volume (a symmetry organized around the center windows within the 19:10 vertical rectangular field), the second overall interval reading is defined fundamentally by a similar boundary of *asymmetry*, which sees attention shift to the edge, as defined by the north facade. This shift of focus occurs if the balconies that project from the front facade, rather than the entrance canopy, define the northern boundary. The balconies, which are cantilevered an additional module beyond the cantilevered strip, define what I call the



4. Schematic diagram of the Villa de Monzie/Stein's west elevation.



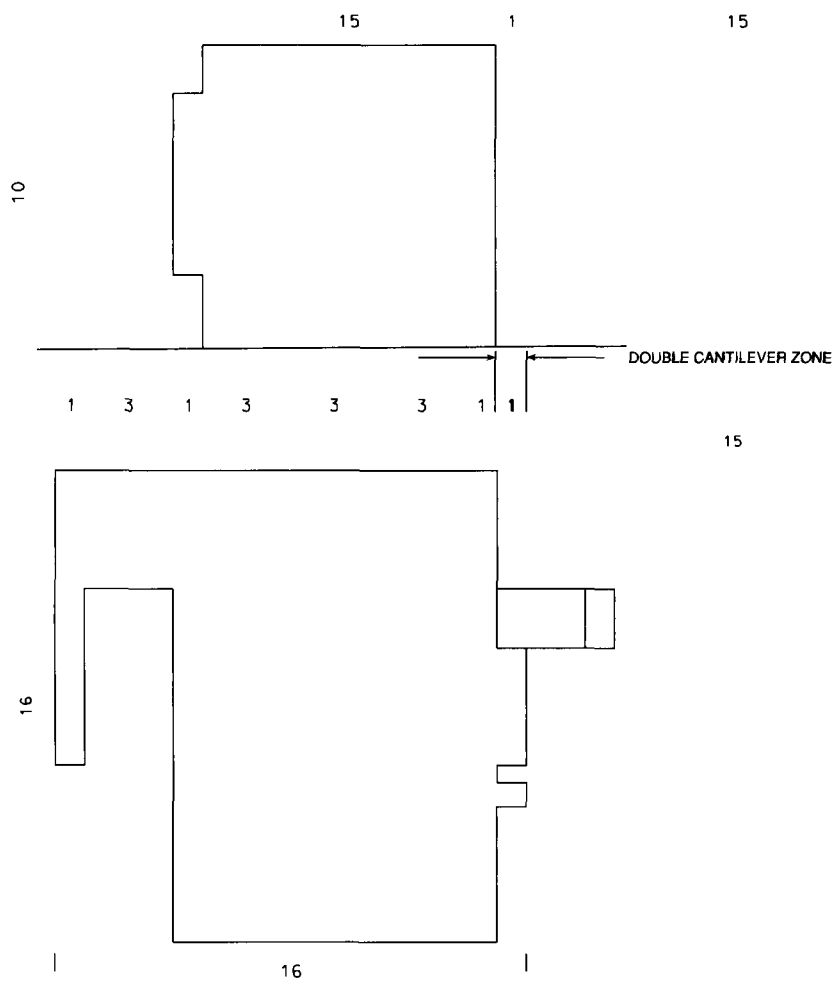
5. Schematic diagrams of the Villa de Monzie/Stein's east elevation and plan.



6. Schematic diagram of the Villa de Monzie/Stein's east elevation, showing the double cantilever zone.

double cantilever zone. Thus, while the boundaries of the plan define a rectangle of elusive significance, whose ratio of depth to width is 19:16 (unless resolved to 20:16, as described above), the double cantilever zone establishes an alternate outer boundary of what is revealed to be the inherent 16:16 square of the composition. In section, these same boundaries define a golden rectangle, 16 moduli wide by 10 moduli high (Figure 6). The module of the double cantilever zone is the dominant interval of the 16 moduli that proceed from south to north (at the east elevation) in the sequence 1:3-1:3:3:3:1-1. The diagram shows how Le Corbusier used the geometry of this 16:10 rectangular field to organize additional elevational moves. The centerline of this asymmetrical field coincides with the south edge of the central windows, creating the basic subdivision of 8 moduli to the south, which is predominantly "void," and 8 moduli to the north, which is predominantly "solid."

While the double cantilever zone establishes the significant edge of this essentially centrifugal reading of the elevation, it also establishes a new geometric center of an opposing centripetal reading. This new center is between the solid of the house and the void of the threshold space in front of the house, as defined by the boundary of the parking court. The double cantilever zone is the zone of overlap that defines a double square in plan and a double golden rectangle in section⁴⁷ (Figure 7). In plan, therefore, the boundaries of the horizontal field have now been expanded to a rectangle 31 moduli by 16 moduli (31:16). In elevation, the boundaries of the vertical field have been expanded to a rectangle 31 moduli by 10 moduli (31:10). Thus the intervals that define the reading from south to north (in plan and elevation) along the sequence of 31 moduli are as follows: 1:3-1:3:3:3:1-1-15. Other than the ground-floor entrance terrace, which is a two-dimensional surface, the entrance canopy is the one element that penetrates the double cantilever zone and bridges the two realms.



7. Schematic diagrams of the Villa de Monzie/Stein's east elevation and plan, showing the figural and proportional interlock of the villa ("solid") and entry court ("void").

Text & Drawings/Photographs

On another level, though graphic, narrative, and mathematical information relating to the end elevations of the Villa de Monzie/Stein is conspicuously missing in Le Corbusier's *Oeuvre complète*, one is inclined to see in his selection of photographs the implied significant presence of the cantilever at every turn, including the view of the interior behind the north facade (Figure 8). Especially significant is the juxtaposition of the two photographs that introduce the chapter.⁴⁸ If the left (here, Figure 9) declares the calm authority of the contracted frontal plane, organized according to the stable alternation of east-west intervals, the right countervails: It proclaims the dynamic, disruptive power of the cantilever/double cantilever in its various expressions, and the latent authority of the north-south section (whose presence is also implied by the deep space of the left photograph) from which it originates. Here Le Corbusier sets the stage for a dialectical drama that is continued in the ensuing pages of the chapter on the villa at Garches in the *Oeuvre complète, 1910–1929*, between text and drawings on the one hand and photographs on the other. Whereas the primacy of the

east-west intervals is manifested unequivocally in the former, by contrast, the significance of the north-south intervals, which is all but unacknowledged in text and drawings, asserts itself in the photographic record. As if to underscore the odd, almost mischievous balance that obtains, and thereby to hint at where an essential key to the ideal mathematics that governs the project ultimately is to be found, in these two initial photographs, Le Corbusier made the image on the right three times larger than the one on the left.

3: Site

In this case, mathematics provide some comforting truths: one leaves one's work with the certitude that the exact result has been reached.

—Le Corbusier⁴⁹

The Extended Field

Le Corbusier's preoccupation with the idea of mathematics as the "cabalistic key"⁵⁰ by which an architecture may be locked into the



8. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. View of the interior. Note the spatial significance of the cantilever zone—the villa's mathematical datum, 1—at the north facade (right). (From *Oeuvre complète, 1910–1929*, p. 148. Courtesy of Fondation Le Corbusier.)



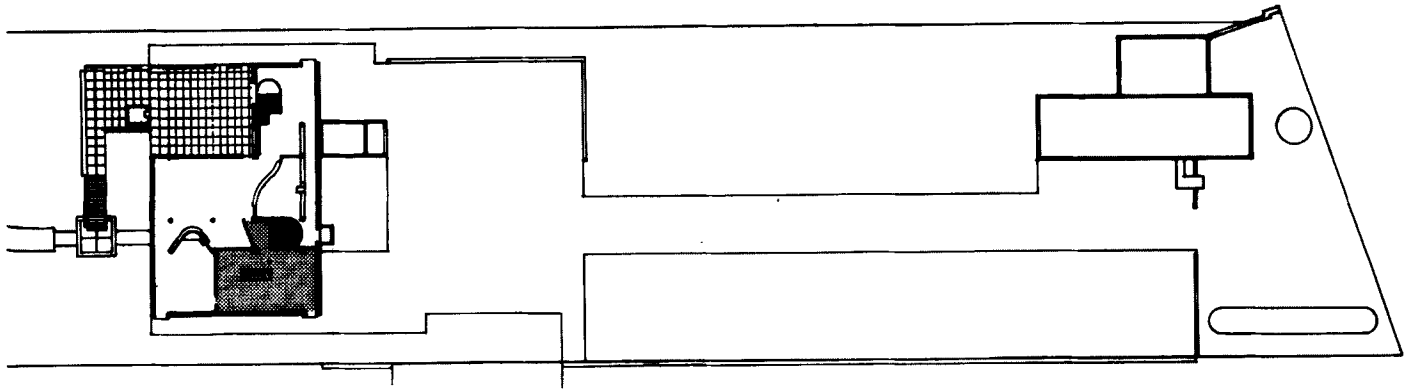
9. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. View of the site threshold. Note the "deep space | collapsed space"—the "outdoor room"—defined by the east-west surface of the villa's north facade and the north-south volume of the gatehouse. (From *Oeuvre complète, 1910–1929*, p. 141. Courtesy of Fondation Le Corbusier.)

harmony of the cosmos makes itself manifest on other levels at the Villa de Monzie/Stein. It appears that he may well have employed the 1:2:3:4-based ratio system to regulate the north-south intervals of the deep space of the entire site, including the important relationship of the villa (main house) to the gatehouse.

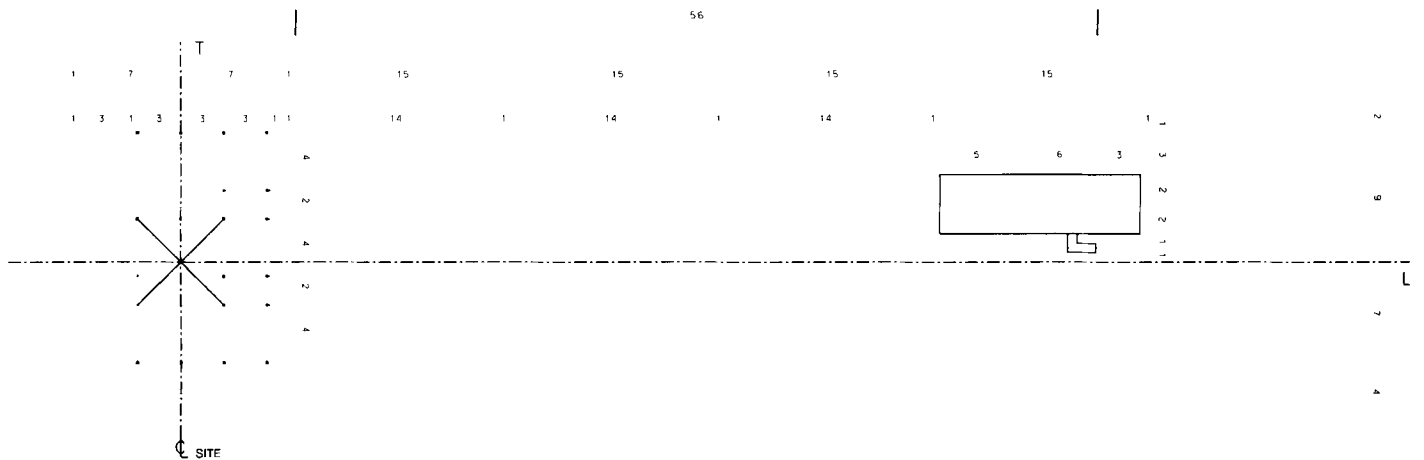
Using Werner Seligmann's field measurements and field-measured site plan as my point of departure (Figure 10), I have conjectured an idealized set of relationships⁵¹ (Figure 11). A number of things may be observed. First, Le Corbusier employs the device of displacement—effected through parallel axes and lateral and diagonal shift—to establish a spatial tension between the center of the villa and the center of the site, thus revealing their interrelationship to be the result of rigorous calculation. The villa's north-south centerline, which is also the centerline of the gatehouse portico, is shifted one module to the west relative to the longitudinal site centerline, which falls at the three-quarter point of the four-module central bay. (Le Corbusier marks this by a subtle secondary move on the facade.) If the villa's east-west centerline is also conjectured to have been similarly shifted relative to the transverse site centerline,⁵² in this case, one and a half modules to the north and thus falling along an actual structural line, then this double shift may be seen to establish a strategically located six-module (4+2, east-west) by six-module (3+3, north-south) x that rather clearly marks the center of the site. This x —a 6:6 square whose boundaries are spatially significant within the organization of the *piano nobile* plan—establishes a dynamic, precisely delimited asymmetrical condition of "center" within the interior of the villa.

Second, the concomitant issue of the inequality between the villa's east and west property-line setbacks is also the result of obvious mathematical calculation. Although the actual width of the site is dimensionally insufficient to realize the intention completely (and zoning requirements complicated the matter),⁵³ the 4 and 2 intervals seem clearly intended to extend beyond the villa's end walls to define these setbacks as well.

Third, another example of the authority of the longitudinal site centerline to organize important physical phenomena within the matrix of the site is seen in its colinearity with the west edge of the driveway. This provides another example of the thematic play of the "edgeline" opposition intrinsic to the project. It also signals Le Corbusier's use of the device of a "diptych-line" to organize the left and right "formal-spatial" and processional events of the villa and its site (for example, the solid form of the gatehouse to the west of the longitudinal site centerline is counterbalanced by the void-space of the entry threshold to the east; the driveway and the servants' entrance, which are axially aligned, are confined to the east



10. Drawing by Werner Seligmann, based on his field measurements, of the Villa de Monzie/Stein's partial site plan, showing the *piano nobile* level of the villa. (Seligmann's drawing, though it does not show the entire site, includes slightly more of the southern part of the site than is included here.) (Courtesy of Werner Seligmann.)



11. Diagram of the Villa de Monzie/Stein's partial site plan (*piano nobile* level of the villa), showing the idealized mathematics of the "extended field." (Based on Seligmann's field-measured site plan and Le Corbusier's site plan studies published in *LCA*, vol. 3.)

side, but the longitudinal site centerline must be crossed in order to move to the front door, which is on the west side).

Fourth, in addition to the site's arithmetical logic—the ratios that underlie the relationship of villa and entry court form the basis of the site's overall metering system (Figure 7)—the same logic has been applied to the ratios that govern the size and positioning of the gatehouse within this system. The villa and the gatehouse are numerically, and thus dimensionally, interrelated through the same simple integers, with the addition of the numbers 5 and 6 contributing to the simultaneous assertion of the independent identity of the gatehouse.

Figure/Field

The arithmetical rigor of Le Corbusier's site plan underscores the axial and figural relationships between the villa and the gatehouse. Seligmann's drawing confirms the clear relationship of axial centerlines that correlates the two buildings and sustains their formal interaction. Moreover, Le Corbusier was obviously as concerned with the space between these two solid forms as with the

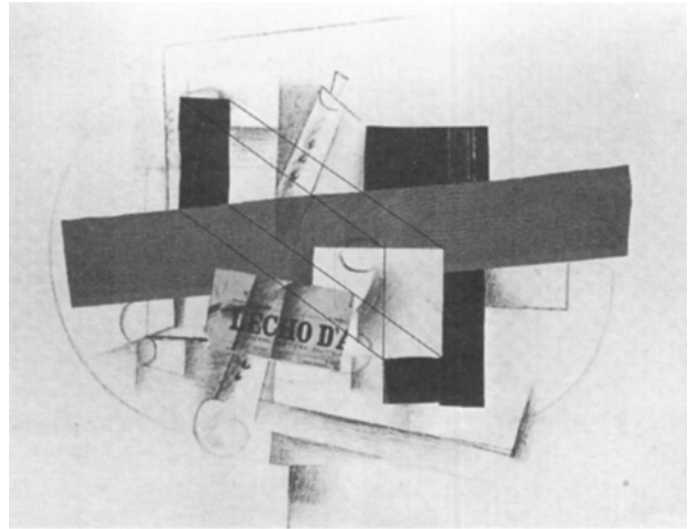
forms themselves. That is, he was as concerned with the algebra of the empty space—of the "outdoor room," or void—defined by the two objects as with the objects themselves. Within Le Corbusier's disciplined chess game delimiting solid and void, affinities between figure and field operate on many levels. A similar ratio in plan (7:2) governs the small "figure" of the principal rectangular block of the gatehouse (14:4) and the "field" of the large space that extends to the gatehouse portico from the villa's front facade (56:16). Of the two volumes that comprise the gatehouse, the subordinate side volume is a 4:6 rectangle in plan and may be thus easily understood as a reciprocal figure to the void of the villa's southwest terrace (this void is in lateral alignment with the villa's 6:6 square site center described above). There are various readings, such as the one shown here (Figure 12), by which the principal volume of the gatehouse, 14:4, may also be conceptualized as a fragment that has been subtracted from the solid volume of the villa. This idea, whereby the gatehouse is viewed as a cutout, separated from the villa and displaced on the site—the device I call the *breakaway*—is one that Le Corbusier learned from post-1911 Synthetic Cubist experiments,

12. Diagrams by the author of the Villa de Monzie/Stein's site plan, showing the "solid | void" ("figure | field") relationships, mathematically correlated (see Figure 11), between the villa and the gatehouse.

such as those by Georges Braque (Figure 13), who was equally mathematically minded in his organization of the visual field.⁵⁴ For all its apparent autonomy, the Villa de Monzie/Stein, within its adopted aesthetic system of formal moves, is as much about relationships as it is about objects. The villa and gatehouse are two elements of one site, interdependent fragments of a larger whole, whose "figure|field" counterpoints and interlocks are mathematically disciplined through the syncopated rhythms of simple whole-number relationships.

SIMILIXI⁵⁵

The positioning of the house at the center of the site causes it to function as diptychlike mediator between front and back landscapes. Le Corbusier believed that "the exterior is always an interior."⁵⁶ This gives rise to a reading of the Villa de Monzie/Stein in which the entire site becomes "building" (and the entire building, "site")—a "building" that is primarily a void and whose roof is therefore principally celestial. In this "site-as-building," the front (northern) and back (southern) landscapes are the two biggest rooms. If the southern landscape is the *extra large* outdoor room, by implication extending infinitely to the horizon, then the northern landscape—the "entry hall" to the villa from the street—is the compressed *large* outdoor room. The villa's roof terrace functions as the *medium* outdoor room, and its *piano nobile* south terrace as the *small* one (Figure 22): four outdoor rooms, four "gardens." Thus the Villa de Monzie/Stein Garches presents a clear homologous relationship between the mathematical and conceptual ordering of space inside and outside, and the idea of *four*—a sequence of four numbers, 1:2:3:4, and four scales, S:M:L:XL—is an important part of its poetic logic.



13. Georges Braque, *Clarinet*, 1913. Black lines, added by the author, show the "solid | void" displacement or "breakaway move"—a formal | spatial device that underlies the interlocking relationship between villa and gatehouse at the Villa de Monzie/Stein.

1:11:2

Le Corbusier's mathematical regulation of the site heightens perception of the underlying devices of symmetry and asymmetry, which represent the formal|spatial expression of the more general dialectical concept "sameness|difference." The ratio 1:1 is the ratio of symmetry, of equality. Relationships that are 1:1 are based on division into halves. The 1:1 ratio represents the equal or diptychal condition of the split screen.⁵⁷ It is the underlying principle of "centering." The ratio 2:1 or 4:2, Le Corbusier's favorite proportional device, is the fundamental ratio of asymmetry, of inequality. Relationships that are 2:1 are based on division into thirds. The 2:1 ratio represents the basic unequal condition of the split screen. It is the underlying principle of "decentering." Ratios that are 3:1 (division into quarters), 4:1 (division into fifths), and so on, extend the series (which is infinite) of clearly definable conditions of asymmetry and attendant split screens. These are the ratios that are at play in Le Corbusier's masterful control of the complex symmetries and asymmetries—the complex centerings and decenterings—that inform the composition of the architectural fields of the Villa de Monzie/Stein.

4: Main Facades

All the elements of the façade are in harmony with one another. Precision has created something final, sharp, true, unchangeable and permanent, which is the architectural moment.

—Le Corbusier⁵⁸

Module|Golden Number

Finally, attention may be directed anew to the issue of Le Corbusier's regulation of the Villa de Monzie/Stein's north and south facades. It is generally recognized that these facades represent

Le Corbusier's early attempt to posit a dualistic mathematical proportional system. In its mature stage, this system, as Wittkower and others have observed, formed the basis for Le Corbusier's *Modulor* system, which he published in 1948.⁵⁹ Fundamentally, the system relied on ideas that are revealed in the renowned drawings of the villa's facades (Figure 2). In these drawings, Le Corbusier asserted the combined principles of the "module," the "golden number," and the "regulating line." The first two principles were important during the design of the project. The diagonal regulating lines were added after the project was designed.⁶⁰ Moreover, in his use of the golden number, Le Corbusier suppressed its essential incommensurable identity (1.618 . . .) and emphasized its rationalized commensurable approximation, 8:5 (1.6). This enabled the two antithetical ideas—the module and the golden number—to be harmoniously combined. I maintain, therefore, that the mathematical substructure of the Villa de Monzie/Stein is based principally, if not exclusively, on whole-number relationships. By using the device of the golden number approximation, Le Corbusier was able to ensure that the moduli were combined in such a way so as to be proportionally, and therefore culturally, significant on a larger level. Thus the moduli not only have their own intrinsic numerical significance (the first four integers 1,2,3,4), but also combine to form, approximately, the proportions of a geometrical figure (a golden rectangle, both as three-dimensional solid form and as two-dimensional planar surface) that has its own universally resonant, honorific meaning.

Wittkower identifies the larger philosophical significance of Le Corbusier's dialectical system, which his pre-*Modulor* research at the villa at Garches represents: "Two different classes of proportion, both derived from the Pythagorean-Platonic world of ideas, were used during the long history of European art. . . . The Middle Ages favoured Pythagorean-Platonic geometry, while the Renaissance and Classical periods preferred the numerical, i.e., the arithmetical side of that tradition."⁶¹ The "arithmetical/geometrical" dialectic was central to the philosophical structure of Le Corbusier's mathematical logic. At Garches, it is given simplified expression through his rationalized, whole-number "module/golden number" system. Moreover, in the end, the theme of the dialectic, on a deeper level, extends to the role of the module itself. For it is the dual identity of the module—operating simultaneously and interdependently as part of two 1:2:3:4-based ratio systems as well as in conjunction with the golden number's whole-number approximation (16:10 in my numbering system)—that forms the basis for the composition of the Villa de Monzie/Stein's facades. A reading of Le Corbusier's own comments on the villa together with a close examination of his drawings of the facades bring this into focus.

Empty/Full

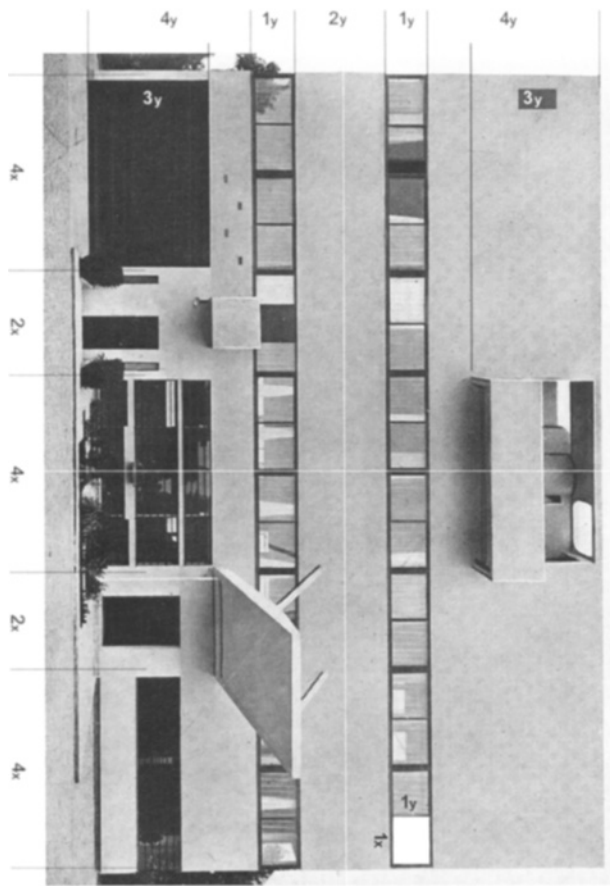
Le Corbusier liked to use the terms *empty* and *full* to describe the phenomenon of alternating "void/solid" intervals of such things as windows ("empty") and wall surface ("full").⁶² In fact, he used these terms to describe the facades at Garches:

Consider drawing 54 [south facade] with the details of the proportions of the villa at Garches. The *choice of proportions*, of full and empty, the determination of the height with respect to a length which in turn is dictated by the constraints of the terrain, all these are in the domain of lyrical creation. . . . However, the mind, curious and grasping, tries to get to the heart of this unrefined product in which the destiny of the work is already permanently inscribed. This search by the mind and the improvements which result from it give rise to the establishment of a mathematical order (arithmetical or geometrical) based on the "golden number," on the interplay of the perpendicular diagonals, on *arithmetical relationships involving 1, 2, 4, between the horizontal bands* [emphasis added], etc. Thus all the elements of the façade are in harmony with one another. Precision has created something final, sharp, true, unchangeable and permanent, which is the architectural moment.⁶³

Le Corbusier maintains that he also used the same "arithmetical relationships" involving the rational geometrical proportion 1:2:4 to determine the heights of the "empty/full" bands of the north facade.⁶⁴ Thus he provokes the conclusion that simple, whole-number ratios regulate more than just the structural/spatial intervals of the plan and *horizontal* rhythm of the facades of the Villa de Monzie/Stein: They also regulate the "empty/full" *vertical* rhythm of the facades. Moreover, not only are relationships involving 1:2:4 at play, but the number 3 is obviously significant as well. In other words, Le Corbusier employed a 1:2:3:4-based ratio system to design his facades (Figures 14 and 15).

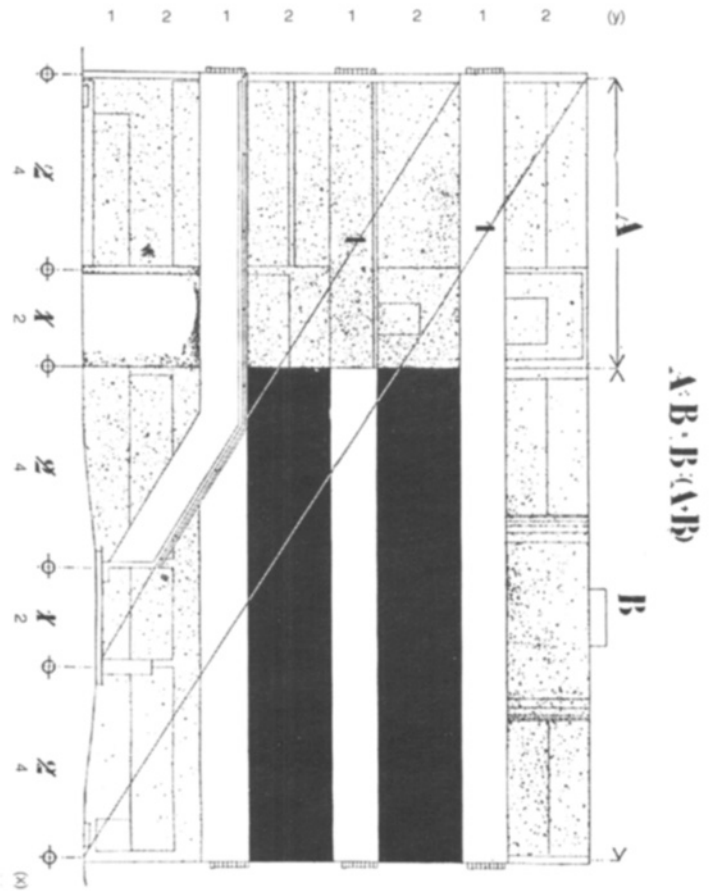
The Fourth "Sign"

A close reading of Le Corbusier's drawing of the south facade, which calls for the decoding of its mathematical "signs," is essential. Up to this point, three separate though ultimately interdependent sets of mathematical "signs" that Le Corbusier included on this drawing have been identified: (1) the numerical proportional system, indicated by the 2:1:2:1:2 ratio sequence, which he employed to regulate the east-west intervals of the structural system and, consequently, the horizontal intervals of the facade; (2) the geometric proportional system of the golden section, indicated by the alphabetical notation *A* and *B*, which he used to determine, through ap-



12y

14. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. View of the north facade. Rotated and marked by the author—and thus abstracted/defamiliarized—showing the idealized mathematical “dependence | independence” of Le Corbusier’s dual 1:2:3:4-based ratio systems that regulate the vertical field ($y = 1$ meter; $x = 1.25$ meters). Note the underlying symmetry. (From *Oeuvre complète, 1910–1929*, p. 147. Courtesy of Fondation Le Corbusier.)



15. Le Corbusier, Villa de Monzie/Stein, Garches, 1927. Drawing of the south facade. Rotated and marked by the author, showing the idealized mathematical “dependence | independence” of Le Corbusier’s dual 1:2:3:4-based ratio systems that regulate the vertical field ($y = 1$ meter; $x = 1.25$ meters). (From *Oeuvre Complète, 1910–1929*, p. 144. Courtesy of Fondation Le Corbusier.)

proximate whole-number relationships, the principal rectangular boundaries and subdivisions of the facade’s two-dimensional field; and (3) the diagonal regulating lines, which he imposed on the drawing after the project was designed in order to proclaim their latent aesthetic refinement and authority. There is a fourth “sign”: the two *Is*, through which the diagonal lines cut and which Le Corbusier placed in the “full” space of the floor structure of the upper two floors. This fourth “sign”—this *other* 1 that appears twice—signifies something different from the 1 that is part of the 2:1:2:1:2 ratio sequence lower in the drawing, where 1 is equivalent to 2.50 meters. It also signifies something different from the 1 that is part of my 1:2:3:4 numbering system, in which 1 is equivalent to 1.25 meters. I believe that this other 1 signifies Le Corbusier’s assertion of a dimensional “truth”—of a dimensional datum: 1 meter. It refers to the actual height of each of the horizontal bands to which it is assigned in the drawing. It is the same 1 to which Le Corbusier referred when he wrote about the “arithmetical relation-

ships involving 1, 2, 4, between the horizontal bands.” This fourth “sign”—1 meter—is the base module that governs the vertical subdivisions of the facades through a *second* 1:2:3:4-based ratio system.

The Dual Module: $1x/1y$

The 1 (1 meter) in this second, elevation-related system, $1y:2y:3y:4y$ (vertical intervals of the facades), is therefore different from the 1 (1.25 meters) in the first, plan-related system, $1x:2x:3x:4x$ (horizontal intervals of the facades). Thus, the corresponding idealized ratio sequences, which can be represented by other permutations as well, are as follows: (xly): north facade = $4:2:4:2:4(x) | 4:1:2:1:4(y)$ (Figure 14); south facade = $4:2:4:2:4(x) | 1:2:1:2:1:2:1:2(y)$ (Figure 15). The y-dimension (height) of the villa’s facades, as well as of the villa’s end elevations, is thereby comprised essentially of 4 floors of 3 modules each, equalling 12 modules (12 meters).⁶⁵ Clearly, then, these two ‘x’ and ‘y’ systems are elegantly related mathematically through the simple ratio 5:4. That is, since ‘y’ = 1 meter, and ‘x’ = 1.25 meters,

then $1y = 4/5$ of $1x$, and $1x = 5/4$ of $1y$. That means that $5y = 4x$, or, expressed another way, 5 meters = 4 times 1.25 meters. In other words, Le Corbusier established a *dual system* of 1:2:3:4-based ratios in this villa, and these two systems are proportional to each other by virtue of the next whole number in the sequence, '5'. By employing different 1:2:3:4-based ratio systems for elevation and plan, Le Corbusier has made the double assertion that they are both independent and dependent. He suggests, therefore, as one might expect, given the dialectical habit of his mind, that there exists in the mental, and therefore the visual, structure of the Villa de Monzie/Stein's facades a mathematically elegant quality of 'simplicity/complexity'. Always cognizant of the relation of "sameness/difference," Le Corbusier has, in effect, employed mathematics as a "unified-field" device; that is, he has interconnected the horizontal (plan) and vertical (elevation) fields in a systematic way, without making them the same. Had he used a single 1:2:3:4-based ratio system for elevation and plan, the proportion of each of the sixteen horizontal window units that comprise the *fenêtres en longueur* of the north facade, for example, might well have been square. Inasmuch as his dominant theme is the rectangle, and that "stasis," which the square represents, contradicts the expression of centrifugal extension and horizontality inherent in the anti-neoclassical counterforces of the north facade, he chose instead to use an "imperfect square," one whose ratio is 5:4. Perhaps, in this individual window unit that signifies the scale of the human body, Le Corbusier gives the clearest expression to the villa's *dual module ratio*. Alternately expressed as '1x|1y', 5:4, or, in meters, 1.25:1.00, the dual module of the window sash constitutes an important mathematical "architectural moment." It represents the elemental copresence of the 1:2:3:4-based harmonic systems that underlie the Villa de Monzie/Stein's ideal Pythagorean-Platonic mathematics.

Acknowledgments

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Notes

A preliminary version of the first part of this article appeared in *Proceedings of the Eighty-fifth ACSA Annual Meeting and Technology Conference* (Washington, DC: ACSA Press, 1997), pp. 388–95.

1. Quoted by Daniel Naegele (source unattributed) in *Le Corbusier: Painter and Architect* (Aalborg: Architectural Magazine B, 1995), p. 83.

2. Colin Rowe, "The Mathematics of the Ideal Villa: Palladio and Le Corbusier Compared," *Architectural Review* 101 (Mar. 1947): 101–4. The article has been republished three times. The first was a reprint of the original article, by the same title, in *Le Corbusier in Perspective*, ed. Peter Serenyi (Englewood Cliffs, NJ: Prentice-Hall, 1975), pp. 46–55. This was essentially identical to the original text; however, the number, selection, and coordination of illustrations were different. The second publication was also a reprint of the original article; it appeared as "The Mathematics of the Ideal Villa," *A + U: Architecture and Urbanism* (Oct. 1975): pp. 29–40 (English text and Japanese translation). Here, the title of the article as well as the composition and scale of the illustrations differed from both earlier editions. The third republication, which comprised a revised text, appeared as the title essay "The Mathematics of the Ideal Villa," in *The Mathematics of the Ideal Villa and Other Essays* (Cambridge, MA: MIT Press, 1976), pp. 1–27. (Since 1978, Rowe's essay has also begun to appear in foreign-language translations.) Although the 1976 edition includes significant revisions to the text and illustrations of the original, the essential thesis remains unaltered. I have noted any discrepancies that are pertinent to my study.

3. Numerous scholars have contributed to the popular association of Rowe with the use of alphabetical nomenclature to describe the east-west structural intervals of Le Corbusier's villa at Garches. In *Modern Architecture: A Critical History* (New York: Oxford University Press, 1980), for example, Kenneth Frampton, referring to Le Corbusier's Villa Schwob and the Villa de Monzie/Stein, writes that "both houses [are] seemingly organized about the classic Palladian ABABA rhythm remarked on by Colin Rowe [emphasis added]," p. 157. In the crucial essay, Rowe does of course associate the villa at Garches with the Palladian model in an indelible way. It is worth remembering that Le Corbusier's connection to Palladio precedes Rowe's observations—as William J.R. Curtis writes, "In the 1930s he [Le Corbusier] told the South African architect, Martienssen, that he tried to recreate "the spirit of Palladio" in the 1920s houses." William J.R. Curtis, *Le Corbusier: Ideas and Forms* (Oxford: Phaidon Press, 1986), pp. 80–84. However, it was other scholars who ascribed to this association the so-called Palladian ABABA designation (see note 6). For example, in his article on "The Grid," *Oppositions* 15–16 (Winter–Spring 1979): 102, Barry Maitland refers to Rowe as his point of departure and employs alphabetical notation to show how the ABABA east-west bay spacing at Garches is a special condition of Rudolf Wittkower's diagram of the general Palladian geometric pattern ABCBA, where C = A. Interestingly enough, however, Wittkower employs no alphabetical designations; see Rudolf Wittkower, *Architectural Principles in the Age of Humanism* (London: Alec Tiranti, 1962; reprint ed., New York: W.W. Norton, 1971), p. 73. Nor, for that matter, does Palladio use them in his own *Quattro libri dell'architettura* (Venice, 1570). In fact, one must turn to Cesare Cesariano's 1521 (Como) edition of Vitruvius's *De architectura* for a systematic use of alphabetical notation to describe grid patterns.

4. Le Corbusier and Pierre Jeanneret, *Oeuvre complète*, 8 vols (Zurich: Editions d'Architecture, 1929–1970; reprint ed., Zurich: Editions d'Architecture, 1967). See the section on the "Villa à Garches 1927," in *Oeuvre complète, 1910–1929*, vol. 1, pp. 140–49.

5. In particular, Rowe described the device of the Greek golden section, especially in the 1976 edition, as an aspect of Le Corbusier's aggressive assertion of

the mathematical organization of the elevations: "Le Corbusier . . . carefully indicates his relationships by an apparatus of regulating lines and figures and by placing on the drawings of his elevations the ratio of the golden section, $A:B = B:(A + B)$." *Mathematics of the Ideal Villa*, p. 9. (See notes 34 and 60.)

6. Analysis of the scholarly record reveals the surprising degree to which a mythology has accrued to Rowe's article as well as to the grid of the Villa de Monzie/Stein at Garches itself. For example, in *The Villas of Le Corbusier: 1920–1930* (New Haven, CT: Yale University Press, 1987), Tim Benton writes, "Ever since Colin Rowe's article 'The Mathematics of the Ideal Villa,' attention has been focused on one aspect of the design [of the villa at Garches] which has been prioritized to the exclusion of other important and salient features. Rowe emphasized the ABABA grid of the Villa Stein-de Monzie, comparing it to a similar grid underlying the design of the villa Malcontenta [emphasis added]," p. 165. First, as I have said (see note 3), Rowe did not employ the formulation ABABA; second, ABABA is not a "grid," and it can only describe a grid if the intervals are the same in both directions, which at Garches they are not; third, Benton's assertion actually functions as an example of how other scholars, but not Rowe himself, have "emphasized the ABABA" (east-west) intervals of the grid but have lost sight of the north-south (what I would call correlatively the CDDDC) intervals—in other words, have lost sight of the grid itself; and finally, the scholarly record reveals a different picture in regard to Benton's suggestion that the grid has received more than its share of attention. Excellent, original research on various aspects of the villa's grid has been published, not the least by Benton himself (see note 10); however, while Rowe is invariably referenced in texts on the subject, in point of fact themes other than the grid and the mathematical have been the primary focus of scholarship. However, Benton is correct in one respect: Inasmuch as Rowe's essay has been accepted as definitive, when scholars have referred to the grid of the Villa de Monzie/Stein, the discussion has usually been limited to attempts to summarize Rowe's observations, focusing almost exclusively and narrowly on the iconic 2:1:2:1:2 (ABABA) comparison with Palladio's Villa Malcontenta.

7. Rowe, *Mathematics of the Ideal Villa*, p. 2.

8. Russian formalism is central to the critical apparatus underpinning my research—namely, the desire to bring a new optic to bear on the familiar object through the technique of *defamiliarization*. Among the seminal texts on the subject is Victor Shklovsky's canonical essay of 1917, "Art as Device/Technique," which stands as the intellectual cornerstone to my approach. See the English edition, translated and edited by Lee T. Lemon and Marion J. Reis in *Russian Formalist Criticism: Four Essays* (Lincoln: University of Nebraska Press, 1965).

9. I have oriented the plans and diagrams that accompany this article with north to the right. This achieves the effect of heightening perception of the north-south intervals, since they are the ones that are therefore read from side to side, the privileged direction. Moreover, this orientation is consistent with Le Corbusier's typical orientation of the Villa de Monzie/Stein site plan drawings, as reproduced in *The Le Corbusier Archive*, ed. H. Allen Brooks, 32 vols. (New York: Garland; Paris: Fondation Le Corbusier, 1982–85), hereafter referred to as *LCA*, with drawings from the Fondation Le Corbusier hereafter referred to as *FLC* with relevant inventory numbers. See, for example, *LCA*, vol. 3, *Le Corbusier: Palais de la Société des Nations, Villa Les Terrasses, and Other Buildings and Projects, 1926–1927*, FLC 10411, p. 370, and FLC 10565, p. 444 (here, Figures 16 and 17). Orientation of the plans and diagrams with north to the right establishes the site's longitudinal (north-south) axis as the horizontal or x-axis (abscissa) of the grid and the site's transverse (east-west) axis as the vertical or y-axis (ordinate) of the grid.

10. Other commentators have addressed historical, morphological, and constructional aspects of the Garches grid. At least three studies have diligently traced

the historical development of Le Corbusier's design of the grid. See Benton, *Villas of Le Corbusier*, pp. 164–89; Arjan Hebly, "The five Points and Form," in *Raumplan and Plan Libre: Adolf Loos and Le Corbusier, 1919–1930*, ed. Max Risselada (New York: Rizzoli, 1988; reprint ed., 1991), pp. 47–53; and Mark Dubois, "2 into 1," *Architectural Review*, CLXXI/1079 (Jan. 1987): 33–36. Barry Maitland's morphological analysis stands out as one of the unique attempts to examine the grid on a purely theoretical level; see Maitland, "The Grid," pp. 90–117. Attention has been devoted to the related mathematical problem of "regulating lines," which, according to Le Corbusier, govern the principal facades. In particular, see Roger Herz-Fischler, "Le Corbusier's 'Regulating Lines' for the Villa at Garches (1927) and Other Early Works," *Journal of the Society of Architectural Historians* 43/1 (March 1984): 53–59. An attempt has also been made to relate the regulating lines to the plan in various diagrams. See Yucuru Tominaga and Shigetaka Nagao, "Rediscovery of Modern Housing/Villa à Garches, 1927," in *Space Design: A Monthly Journal of Art and Architecture* 133 (Sept. 1975): 66–71. Most relevant to the present study, perhaps, is Herz-Fischler's assertion, which is significant on many levels: "It is clear from the documents . . . that Le Corbusier did not hesitate to change his writings or drawings, after the fact, to accommodate his constantly changing views and systems. A consequence of this is that no serious study of Le Corbusier can be based on the 'official' versions alone. In particular, this is the case for Garches [emphasis added]." Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 57. (See note 60.) For a cogent discussion of the constructional aspects of the Dom-ino reinforced concrete frame system—the system employed at Villa de Monzie/Stein, of which the grid is an integral part and principal manifestation—see Eleanor Gregh, "The Dom-ino Idea," *Oppositions* 15–16 (Winter–Spring 1979): 60–87.

11. Rowe, *Mathematics of the Ideal Villa*, p. 8.

12. As published in "Mathematics of the Ideal Villa," p. 103, Rowe's 1947 diagram of the plan of the villa at Garches is oriented with north up. As published in *Mathematics of the Ideal Villa*, p. 5, Figure 1, Rowe's 1976 diagram of the plan is oriented with north to the right. Rowe informed me that this latter orientation was the result of an arbitrary layout decision made by the publisher; for reasons stated above (see note 9), this is the way I have oriented re-creations of both of his diagrams in this article. The typographical error in Rowe's 1947 diagram, which mistakenly assigns the value "1" to the middle north-south bay, was corrected in the 1976 version.

13. The correlative issue that this difference raises, which involves the dialectic between fact and implication, is discussed in note 30.

14. Herz-Fischler asserts that Le Corbusier, in his article "Tracés régulateurs," *L'Architecture Vivante* (spring–summer 1929), 12–24, "tells how the supporting frame gives a 2-1-2-1-2 cadence to the villa at Garches. This is referred to as an 'automatic system of proportioning' (*tracé que j'appellerai automatique*) and was used at Maison Cook, Pessac, and House C-2 at Stuttgart." Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 57.

15. Rudolf Wittkower, "Le Corbusier's Modulor," in *In the Footsteps of Le Corbusier*, ed. Carlo Palazzolo and Riccardo Vio (New York: Rizzoli, 1991), p. 13.

16. The extent to which Le Corbusier himself privileges the east-west, in contradistinction to his silence on the north-south, is reflected in this statement from the introduction to the section on Garches in his *Oeuvre complète, 1910–1929*: "La Maison est entièrement supportée par des poteaux disposés à équidistance de 5 m et 2 m 5 sans souci du plan intérieur" [The house is supported entirely by columns that are placed at distances equal to 5 meters and 2.5 meters throughout the interior] (my translation), p. 140. Le Corbusier neglects to clarify that this is true only with respect to the east-west spacing; therefore, he implies that the 5- and 2.5-meter intervals are used to determine the north-south placement of the columns as well, which, of course, is not the case. (See note 39.)

17. Empirical evidence has helped to corroborate this. In the course of conducting my research, I have observed that both students and faculty colleagues recall accurately either the ABABA or 2:1:2:1:2 aspect of the grid, yet struggle in vain to recall correctly the north-south dimension.

18. When the subject of the grid of the villa at Garches has come up for discussion with students and colleagues over the years, no one has independently raised the idea of *doubling* that is advanced here. Nor, according to my study of the published record, has this device been previously noted in print. I made the discovery in 1989 while teaching at the University of Texas, Arlington, where I conducted a graduate studio that involved an addition to the villa at Garches.

19. Rowe obviously delimits the projection of the terrace in his analytic diagrams to reinforce the comparison with the portico of the Villa Malcontenta, but he leaves the door open to confusion. The fact that the terrace actually extends an additional .5 (½) interval in the Le Corbusier-Rowe numbering system (this half interval also defines the zone of the stair that descends to the garden) appears to be an underappreciated point by many who have reprinted Rowe's diagrams. Among the authors whose discussions I have studied, only Kenneth Frampton aligns Rowe's diagram and Le Corbusier's plan in a manner that accurately and unambiguously indicates that the diagram does not include this additional important zone. Kenneth Frampton, "Frontality vs. Rotation," *Five Architects: Eisenman, Graves, Gwathmey, Hejduk, Meier* (New York: Wittenborn, 1972; reprint, New York: Oxford University Press, 1975), p. 11; and *Modern Architecture: A Critical History*, p. 157. In another article, however, Frampton provides an example of the more common phenomenon, wherein the juxtaposition of diagram and plan is ambiguous and misleading; see "Le Corbusier and 'l'Esprit Nouveau,'" *Oppositions* 15–16 (winter–spring 1979): 41.

20. My diagrams seek to represent the structural condition of the ground floor and *piano nobile* plans, based on Le Corbusier's drawings reproduced in *LCA*, vol. 3. See, for example, ground floor plans FLC 10576, p. 452; FLC 10431, p. 382; and FLC 10451, p. 393. In the same volume, see *piano nobile* plans FLC 10563, p. 443, and FLC 10452, p. 393. The aforementioned plates are conclusive with respect to column locations; however, they exhibit ambiguities with respect to column shapes, especially in the case of the inside two columns at the service entrance on the ground floor and their expressions at the *piano nobile*.

21. To my knowledge, only James Michael Ward, in his doctoral dissertation, "Le Corbusier's Villa 'Les Terrasses' and the International Style" (New York University, 1984), has observed that the house has, as he writes, "thirty-one vertical supports," which agrees with my decipherment of Le Corbusier's ground floor plans reproduced in *LCA*, vol. 3 (see note 20); see Ward, "Le Corbusier's Villa 'Les Terrasses,'" pp. 178; 207, note 45. However, Ward does not comment on the most stunning implication of this discovery, which is that Le Corbusier, by whatever mix of practical necessity and calculated artifice, violates the dictates of his idealized Dom-ino system and presents only the illusion of a cantilevered *piano nobile* at the north facade (the absence of columns at the *fenêtres en longueur* merely signifies that the floor above is cantilevered); at the south facade, however, by way of physical and conceptual opposition, the reality of the structural act of the cantilever at the *piano nobile* is unequivocally presented. It should be noted as well that Ward's research uncovers the surprising fact that it was Gabrielle de Monzie, cotenant with Sarah and Michael Stein, who held legal title to the land and assumed principal fiduciary responsibility for the construction of the villa (pp. 19, 53–56). Villa de Monzie/Stein, therefore, as opposed to Villa Stein or even Villa Stein-de Monzie (though this is how the house is denominated in *LCA*), perhaps more properly conveys the historical record. Le Corbusier evidently named the villa Les Terrasses or The Terraces, though he does not refer to it as such in the *Oeuvre complète*. The drawings, other than the earliest few, which are stamped "Stein de Monzie," bear the desig-

nation "Mme G de Monzie," indicating that Le Corbusier clearly knew who was paying the bills.

22. Wittkower, "Le Corbusier's Modulor," p. 12; see Rowe, *Mathematics of the Ideal Villa*, p. 17, note 6, where Rowe explicitly attributes his observations on the relationship between mathematics, musical harmony, and ideal proportion to his reading of Wittkower.

23. Rowe, *Mathematics of the Ideal Villa*, p. 8.

24. Rudolf Wittkower, *Idea and Image: Studies in the Italian Renaissance* (New York: Thames and Hudson, 1978), p. 110.

25. *Ibid.*

26. *Ibid.*, p. 111.

27. *Ibid.*, p. 111.

28. *Ibid.*, p. 110.

29. *Ibid.*, p. 110. Wittkower also writes, "I do not suggest that Palladio or any other Renaissance artist translated musical into visual proportions; but they regarded the consonant intervals of the musical scale as the audible proofs for the beauty of the ratios of the small whole numbers" (p. 112).

30. Peter Eisenman, in "Aspects of Modernism: Maison Dom-ino and the Self-Referential Sign," *Oppositions* 15–16 (winter–spring 1979): 118–29, analyzes the general theoretical aspects of the issue of extension versus stasis (compression) implied by Le Corbusier's Dom-ino reinforced concrete frame system, of which the villa at Garches is a compound version. He observes, quite rightly, that the Dom-ino implies, because of the location of the columns relative to the end of the floor slab, extension parallel to the long sides and stasis parallel to the short ends: "The location of the columns flush on the ends marks an opposition to the setback columns on the sides, and further suggests that the ends of the slab have been cut off, implying the possibility, or former condition, of horizontal extension of the slab on the long axis" (p. 125). On one level, this reading clearly applies to Villa de Monzie/Stein: The east-west intervals imply extension, or growth, and the north-south intervals, because of the setback of the columns that form the cantilever, imply stasis, or completion. However, due to the contingencies of the site, there exists as well a counterreading, one that implies a tension between fact and implication. Because of the shallow space of the site from side to side, there is no physical extension of the east-west intervals; this dimension of the grid may imply expansion, but the fact is that it stops, it is compressed, static. Because of the deep space of the site from front to back, there is physical extension of the north-south intervals; this dimension of the grid may imply stasis, but the fact is that it extends, it is dynamic. For an elucidation of the deep space/shallow space dialectic in Le Corbusier's work, see Thomas Schumacher, "Deep Space/Shallow Space," *Architectural Review* CLXXI/1079 (Jan. 1987): 37–42.

31. For articulation of the influential modernist concept of Significant Form, see Clive Bell, *Art* (Oxford: Oxford University Press, 1914; reprint, New York: Capricorn Books, 1958).

32. Le Corbusier's enthusiasm for the reassuring authority of mathematics was tempered by his ultimate insistence on intuitive control of visual phenomena. On the one hand, he had confidence in what he called the "Q.E.D. of the mathematician"; see Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 58. On the other hand, he extolled through musical metaphor ("instrument") the ineffable faculty of artistic sense, acquired through painting: "The painter's [instrument] is his eye which truly acts as an instrument of control, verification, and penetration" (p. 58). Le Corbusier's faith in the connection between mathematics, painting (visual judgment), and music is interwoven in statements such as this: "*Architectural composition is geometric*, an event primarily of a visual nature; an event implying judgments of quantities, of relationships; the appreciation of proportions. Proportions

provoke sensations; a series of sensations is like the melody in music [Le Corbusier's italics]." Le Corbusier, *Precisions: On the Present State of Architecture and City Planning*, trans. Edith Schreiber Aujame (Cambridge, MA: MIT Press, 1991), p. 133.

33. For Le Corbusier's veneration of the Parthenon, see in particular "Architecture, Pure Creation of the Mind," in Le Corbusier, *Towards a New Architecture*, trans. Frederick Etchells (London: Architectural Press, 1927; reprint, New York: Praeger, 1960), pp. 185–207. Le Corbusier, expressing his reverence in general for "creations of calculation" (p. 212), as evidence of "the higher levels of the mind" (p. 204), asserts that "the Parthenon gives us sure truths and emotion of a superior, mathematical order" (p. 221).

34. Two examples of how Garches is typically understood in terms of the golden section, which can be deduced from study of the plates Le Corbusier published in the *Oeuvre complète*, are as follows: First, as indicated earlier (see note 5), he includes the basic proportional assertion of the golden section on his drawing of the south facade, $A:B = B:(A + B)$, to indicate that he has used this to organize its fundamental left-right ("solid/void") proportional relationship. Thus, according to the east-west intervals along the bottom of his drawing, $A = 2 + 1 = 3$, and $B = 2 + 1 + 2 = 5$. Therefore, the ratio $A:B = 3:5$, and the ratio $B:(A + B) = 5:8$. The actual proportion of the golden section, which can be derived geometrically and arithmetically, is approximately .618 (a unique property of the Greek golden section is that its reciprocal is the same number added to 1—that is, 1.618). This is an irrational number that whole number ratios can only approximate; for example, the ratio of the numbers $3:5 = .6$, and the ratio of the numbers $5:8 = .625$. The ratio $5:8$, which more nearly approximates the actual number, is the most common rationalized (that is, approximate) expression of the golden section, and one that Le Corbusier routinely used. Second, this is seen in the second example of the regulating presence of the golden section, which relates the ratio of the sum of the north-south intervals to the sum of the east-west intervals of the enclosed part of the ground-floor plan (that is, the ratio of the short side of the villa to the long side of the villa). In the Le Corbusier-Rowe system, the sum of the north-south intervals is $\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 5$. The sum of the east-west intervals is $2 + 1 + 2 + 1 + 2 = 8$. Thus their ratio is $5:8$. This means that the enclosed rectangular field of the ground-floor plan conforms to the conventional, idealized (whole number) ratio of the golden section, and, in theory, so does the rectangular vertical field of the north and south facades; the ratio of height to length is also $5:8$. (In my doubled numbering system the ratio is correspondingly doubled and expressed as $10:16$.) Rowe refers to this important approximation of the golden section ($5:8$) when describing the similarities between the volumetric proportions of Palladio's Villa Malcontenta and the Villa de Monzie/Stein at Garches: "Both are blocks of corresponding volume, each measuring 8 units in length, by $5\frac{1}{2}$ in breadth [this is at the *piano nobile*; the ground-floor plan is 5 in breadth], by 5 in height." Rowe, *Mathematics of the Ideal Villa*, p. 4. This is an example of the way in which Rowe's text has reinforced the association of the golden section—its whole number approximation—with the proportional structure of the villa at Garches. (See note 60.)

35. For an explanation of Aristotle's idea of *taxis*, "the orderly arrangement of parts," which includes the double idea of the grid and tripartition, see Alexander Tzonis and Liane Lefaivre, *Classical Architecture: The Poetics of Order* (Cambridge, MA: MIT Press, 1986), p. 9.

36. From an interview with Le Corbusier by Hugues Desalle, transcribed from an audio recording made six months before the architect's death and published in *Modulus/The University of Virginia School of Architecture Review* (1979): 71.

37. Perhaps the Derridean idea of the double session—"displacement without reversal" of inherited concepts and categories"—provides the intellectual lens through which to view this alternative numerical and alphabetical reading of the Villa de Monzie/Stein's grid vis-à-vis Le Corbusier's own "official" view and popu-

lar terminology. See Christopher Norris, "Deconstruction, Post-modernism and the Visual Arts," in Christopher Norris and Andrew Benjamin, *What Is Deconstruction?* (London: St Martin's Press, 1988), p. 31.

38. Rowe, *The Mathematics of the Ideal Villa*, p. 4. Moreover, the significance of the cantilever to the visual and intellectual structure of the villa at Garches is central to Rowe's later essay, written with Robert Slutzky, "Transparency: Literal and Phenomenal," *Perspecta: The Yale Architectural Journal* 8 (1963): 45–54—Max Risselada pursues the theme in "Free Plan versus Free Facade: Villa Savoye and Villa Baizeau Revisited," *Raumplan and Plan Libre*, pp. 55–63. He describes the confrontation of the free plan and free facade within Le Corbusier's projects: "This confrontation takes place in the zone created by the cantilever of the domino structural unit: the strip between the columns and the skin of the facade. It is this strip, which during the formulation of the 'five points,' during the design process of Maison Cook and the villas Meyer and Stein-de Monzie respectively, was allotted an increasingly independent position" (p. 56). According to Benton, Le Corbusier first introduced the cantilevered strip at the front facade only (*Villas of Le Corbusier*, pp. 174–75), which is ironic in light of the structural realities of the actual building (see note 21).

39. In point of fact, Le Corbusier's preliminary drawings indicate that he experimented with just such an idea—at one point in the design process, the intervals were the same. Maitland identifies an early scheme "in which the building is five equal square bays wide" and principally two square bays deep. Maitland, "The Grid," pp. 101–2. Benton points out that Le Corbusier also experimented with an ABABA rhythm in both dimensions of the grid. Benton, *Villas of Le Corbusier*, p. 170. This condition is clearly expressed in Le Corbusier's site plan FLC 10411, *LCA*, vol. 3, p. 370 (here, Figure 16).

40. Rowe, "Mathematics of the Ideal Villa," pp. 102, 103, captions to the diagrams of the grids of the Villa Malcontenta and the villa at Garches.

41. "Standard framework of Le Corbusier's Domino House, designed in 1914, for mass-production, which shows how the quality of partial paralysis inherent in the plan of solid wall buildings such as Palladio's Villa Malcontenta is, in reinforced concrete structure, transferred to the section." Rowe, "Mathematics of the Ideal Villa," p. 101.

42. In her summary of the architectural potential that may be attributed to the Dom-ino system, Gregh highlights the property of the "module," explaining that "the reduction of the building to a few standardized elements provides the basis for systems of modular proportion." Gregh, "The Dom-ino Idea," p. 61. She notes, moreover, that this is "the first advantage noted by Jeanneret in his notes for the patent, in Sketchbook 1915–16, p. 58 (note 1)," (p. 79, note 2). Gregh suggests that the idea of the geometric module inherent in Le Corbusier's 1914 Dom-ino is ultimately the result of his dialectical mind, simultaneously empirical (practical) and Platonic (ideal): That is, it results from a conflation of the demands and benefits of industrial standardization (the economy and efficiency of standardized elements and measurements) on the one hand, and the metaphysical demands of pure mathematical proportions and rationalization, on the other. According to Gregh, Le Corbusier wanted to create a constructional/architectural system "based on the multiples and divisions of a geometric module, like all great architecture of the past [emphasis added]" (p. 68). On another level, Stanislaus von Moos refers to Joseph Paxton's Crystal Palace as the "birthplace of the 'module' in the architecture of the industrial age," in *Le Corbusier: Elements of a Synthesis* (Cambridge, MA: MIT Press, 1980), p. 310. This observation descends from Wittkower, who asserts that Paxton's structure was the "first . . . of colossal size erected of standardized units over a grid. The logic inherent in the industrial revolution enforced a dimensional order." Wittkower, "Le Corbusier's Modulor," p. 14.

43. For discussion of Le Corbusier's deployment of the 2.5-meter sliding window unit (comprised of two 1.25-meter sashes), see Edward R. Ford, *The Details of Modern Architecture* (Cambridge, MA: MIT Press, 1990), pp. 244–49.

44. Rowe writes that whereas Palladio preferred the triple division, it was "Le Corbusier's propensity to divide by four." Rowe, *Mathematics of the Ideal Villa*, p. 11. In the original text, Rowe gives the example that the facade is divided into four horizontal units (that is, the four floors are clearly expressed at the south facade). Rowe, "Mathematics of the Ideal Villa," p. 103. In addition to the various 1:4 relationships that regulate the window subdivisions of the facades, which at the north effectively countersubdivide the overall 20-meter length into four bays of 5 meters each (each bay is comprised of four horizontal windows), four free-standing columns mark the threshold within the entrance hall, and the site is essentially divided into quadrants. The 1:4 ratio was a favorite of Le Corbusier's throughout his career. For example, he employed it at his mother's house in Vevey, 1923: The parallelepiped is 4 meters wide by 16 meters long.

45. Research by Benton and Hebly reveals the neoclassical preoccupation with "center" that underlies Le Corbusier's final ABABA rhythm of the east-west intervals. Hebly writes, for example, "Le Corbusier's approach suggests a width of 20 metres: 4 times a 5-metre bay. However, the centre is then occupied by a column, inconceivable in the front facade of the classic villa. This is the place for the entrance, certainly not for a column. . . . Indeed, in all the early plans of this last version, the entrance is in the middle. . . . For the proportions of the house this means: one 5-metre bay in the middle, with a remainder of 7.5 metres at either side. These can only be divided into a half-bay of 2.5 metres and one of 5 metres." Hebly, "Five Points and Form," p. 52; see also Benton, *Villas of Le Corbusier*, p. 171.

46. My differentiation between facade and elevation is a result of my correspondence with Rowe. In response to an early draft of this article, he wrote, "I think that your article would gain if, with reference to enclosing vertical planes, you were to introduce the designation *facade* as something distinct from *elevation* because though obviously related, after all these are, in the end, two very different species. So when does an elevation become a facade? And, in these matters, I tend to suppose that, almost always, a facade is a vertical surface endowed with some metaphorical or allegorical presence. Somewhat like the title page of say Palladio's *Quattro Libri*, it may be an announcement of content, but it may also conceal quite as much as it discloses. While in contradistinction, an elevation is a much more literal statement which, on occasion, may serve to convey certain important sectional information. But I also would attribute a privileged status to facade." This portion of the letter was published subsequently in *ANY: Architecture New York Magazine* 7–8 (1994): 5. In point of fact, the differentiation between facade and elevation at the Villa de Monzie/Stein is more complex and equivocal, as the Loosian attributes of the west "elevation" suggest.

47. The site plan by Werner Seligmann is the basis for this observation (see note 51). Seligmann confirmed in conversation that his measurements of the villa at Garches show that the distance in front of the house to the boundary of the parking court is a square, completing a golden rectangle with the ground-floor plan, as his published drawings imply. See Michael Dennis, *Court and Garden: From the French Hotel to the City of Modern Architecture* (Cambridge, MA: MIT Press, 1986), p. 200; and Frampton, "Le Corbusier and l'Esprit Nouveau," p. 37. Dorothee Imbert's study, *The Modernist Garden in France* (New Haven: Yale University Press, 1993), also offers excellent insights on the site of Villa de Monzie/Stein; however, it should be noted that her "reconstituted site plan" (p. 155) is not in accord with Seligmann's on-site measurements with respect to the dimension (and proportional/mathematical implications) of the parking court. The site plan for one of Le Corbusier's early schemes, however, which Imbert reconstructs on the basis of FLC 10411, *LCA*, vol. 3, p. 370 (here, Figure 16), does agree with Seligmann. See Imbert, *Modernist Garden in France*, p. 157.

48. Le Corbusier, *Oeuvre complète, 1910–1929*, p. 141.

49. The statement is specifically about his villa at Garches, in Le Corbusier, *Oeuvre complète, 1910–1929*, p. 144. The translation is by von Moos, *Elements of a Synthesis*, p. 179.

50. *Ibid.*

51. Seligmann graciously provided me with photocopies of his on-site measurements, which, though very difficult to decipher, were critical in confirming the basis of his published partial site plans (see note 47) and in formulating my conjecture.

52. Review of the available documents, such as FLC 10565, *LCA*, vol. 3, p. 444 (here, Figure 17), reveals Le Corbusier's basic intention to position the villa at the center of the site. However, the documents are inconclusive with respect to the villa's micropositioning that determines the *exact* center or slight shifts *off-center* that I conjecture here. Perhaps only through rigorous site reconnaissance, which requires on-site verification of the original property survey and new field measurements, can this important issue be clarified.

53. According to Benton, the site is 26.9 meters wide. Benton, *Villas of Le Corbusier*, p. 166. According to Hebly, it is 27 meters wide. Hebly, "Five Points and Form," p. 51. According to FLC 10447, *LCA*, vol. 3, p. 391, the site is 26.9 meters wide. The *idealized* site plan that I conjecture would require a site swidth of 27.5 meters: The east property line setback is slightly less than the idealization requires in order to obtain a full interval of 4 (5 meters).

54. See, for example, Jeffrey Hildner, "Collage Reading: BraquelPicasso," in *Proceedings of the Eighty-fourth ACSA Annual Meeting and Technology Conference* (Washington, DC: ACSA Press, 1996), pp. 181–87.

55. Rem Koolhaas and Bruce Mau, *S, M, L, XL* (New York: Monacelli Press, 1995).

56. Le Corbusier, *Towards a New Architecture*, p. 191.

57. For an elucidation of the term *split screen* in Le Corbusier's work, see Schumacher, "Deep Space," pp. 37–42.

58. Le Corbusier's description of the Villa de Monzie/Stein is quoted by Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 59. The translation is slightly different in Le Corbusier, *Precisions*, p. 72.

59. Wittkower, *Idea and Image*, p. 122.

60. In the continuation of a passage noted previously (see note 10), Herz-Fischler maintains that "no serious study of Le Corbusier can be based on the 'official' versions alone. In particular, this is the case for Garches; as the various early documents and drawings indicate, Le Corbusier made no use of the golden number in its design [emphasis added]." Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 57. According to Herz-Fischler, Le Corbusier's regulating lines, which are an expression of the incommensurable golden number proportional system, were applied to the facade drawings that appear in the *Oeuvre complète* after the project was completed. On the other hand, von Moos points to "Matila Ghyka, *Esthétique des proportions dans la nature et dans les arts* (Paris, 1927), where Ghyka publishes and discusses some of Le Corbusier's proportion studies, especially the regulating lines of the Villa Stein at Garches." Von Moos, *Elements of a Synthesis*, p. 366, note 98. Thus the very year the villa was completed, Ghyka used it to show how Le Corbusier had employed golden number–regulating lines. However, von Moos also seems to lend indirect support to Herz-Fischler's theory of post facto regulating lines: "There can be no doubt that Ghyka's book [*Le Nombre d'Or*, published in Paris in 1931] played an important role in Le Corbusier's coming to grips with the problem of the golden section in nature and art" (p. 310). One might well conclude that while regulating lines were not part of Le Corbusier's design process at Villa de Monzie/Stein, use of the geometric approximation of the golden section surely was. Le Corbusier's interest in the golden number, both in architecture and in

painting, predates the villa at Garches. In addition, the iconic whole-number approximation of the golden ratio—8:5—is an undeniable property of the villa's basic physical relationships, which surely cannot be explained as a post facto phenomenon (see note 34). Interestingly enough, of the four original illustrations of Rowe's 1947 text, the one of Matila Ghyka's harmonic decompositions of the golden rectangle is the only one that is not included in the 1976 edition; see Rowe, "Mathematics of the Ideal Villa," p. 104. See Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 58, note 9, for insight and commentary on Rowe's omission of this illustration in the 1976 edition and on the issue of the direct influence of Ghyka's book on Le Corbusier's mathematical ideas.

61. Wittkower, *Idea and Image*, p. 116.

62. Herz-Fischler, "Le Corbusier's 'Regulating Lines,'" p. 59.

63. *Ibid.*

64. *Ibid.* p. 57.

65. The idealized height of the villa expressed in whole-number y (1-meter) modules is equal to 12 (12 meters), which equals $9.6 \times$ (1.25-meter) modules. The idealized height of the villa expressed in whole-number x (1.25-meter) modules is equal to 10 (12.5 meters), which equals $12.5 \times$ (1-meter) modules—10 derives from the whole-number approximation of the golden section ratio, 16:10 (see note 34). Both 12 meters and 12.5 meters represent satisfactory approximations of the golden section ratio in relation to the width of the villa, which equals 20 meters (12.5 is slightly more accurate). The documents are inconclusive as to the definitive height of the villa, and given that Le Corbusier's *modus operandi* was to continually make trade-offs between idealizations and empirical demands, one should not be surprised to learn that the villa as detailed in the construction documents or as built may not be in precise accord with either of these two dimensions.